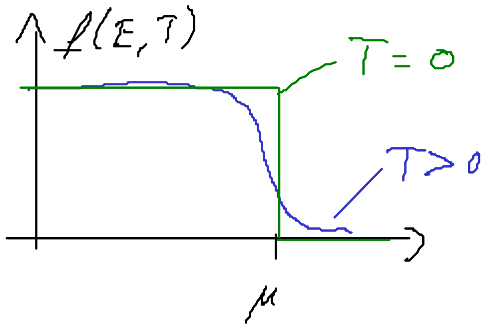
Sommerfeld - Modell



E_F definiert durch $f(E_F, T=0) = \frac{1}{2}$

$$\mu\big|_{T=0} = E_F$$

$$f(E,T) = \frac{1}{e^{\frac{E-\mu}{k_B T}} + 1}$$

$$E_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}$$

$$n = \frac{N}{V} = \int_0^\infty D(E)\, \underbrace{f(E,T)}_{T=0}\, dE = \frac{(2m)^{3/2}}{2\pi^2\hbar^3} \cdot \frac{2E_F^{3/2}}{3}$$

Fermi - Wellenvektor $\quad k_F = (3\pi^2 n)^{1/3}$

Fermi - Impuls $\quad p_F = k_F \hbar$

Fermi - Geschwindigkeit $\quad v_F = \frac{\hbar}{m}(3\pi n)^{1/3}$

Fermi - Temperatur $\quad T_F = \frac{E_F}{k_B}$

	$n\ \left[\frac{10^{28}}{m^3}\right]$	$k_F\ \left[\frac{1}{Å}\right]$	$v_F\ \left[10^6 \frac{m}{s}\right]$	E_F [eV]	T_F
Al	18,1	1,8	2,0	11,7	
Cu	8,5	1,4	1,6	7,0	
Ag	5,3	1,2	1,4	5,5	

für $T \neq 0$: $\quad \mu(T) \approx E_F\left[1 - \frac{\pi^2}{12}\left(\frac{T}{T_F}\right)^2\right]$

spezifische Wärme

$$c_V = \frac{\partial}{\partial T} U(T) = \frac{\partial}{\partial T} \int_0^\infty E \, D(E) \, f(E,T) \, dt$$

innere Energie pro Volumen

$$= \frac{\partial}{\partial T} \frac{1}{2\pi^2} \left(\frac{2m}{\hbar^2}\right)^{3/2} \int_0^\infty \frac{E^{3/2} \, dE}{e^{\frac{E-\mu}{k_B T}} + 1}$$

Näherung $U(T) \approx U_0(0) + \delta U(T)$

$$U(0) = \int_0^{E_F} E \, D(E) \, dE = \frac{3}{2} \frac{n}{E_F^{3/2}} \frac{2}{5} E_F^{5/2} = \frac{3}{5} n E_F$$

$$= \frac{3n}{5} k_B T_F$$

$$D(E) = \frac{3}{2} \frac{n}{E_F} \left(\frac{E}{E_F}\right)^{1/2} = \frac{(2m)^{3/2}}{2\pi^2 \hbar^3} \sqrt{E}$$

$$\delta U(T) \approx \underbrace{n k_B T \frac{T}{T_F}}_{\text{Bruchteil der Elektronen}} = \frac{n k_B}{T_F} T^2$$

die die thermische Energie aufnehmen können

$$c_V \approx \frac{\partial}{\partial T} \delta U(T) = \frac{2 n k_B T}{T_F}$$

Näherungslösung: $U(T) \approx U(0) + \frac{\pi^2}{6} D(E_F) (k_B T)^2$

$$\Rightarrow c_V = \frac{\pi^2}{3} \frac{T}{T_F} \frac{3 n k_B}{2} = \gamma T$$

$$\boxed{\gamma = \frac{\pi^2}{2} \frac{n k_B}{T_F}}$$

Sommerfeld-Konstante

Die gesamte spez. Wärme

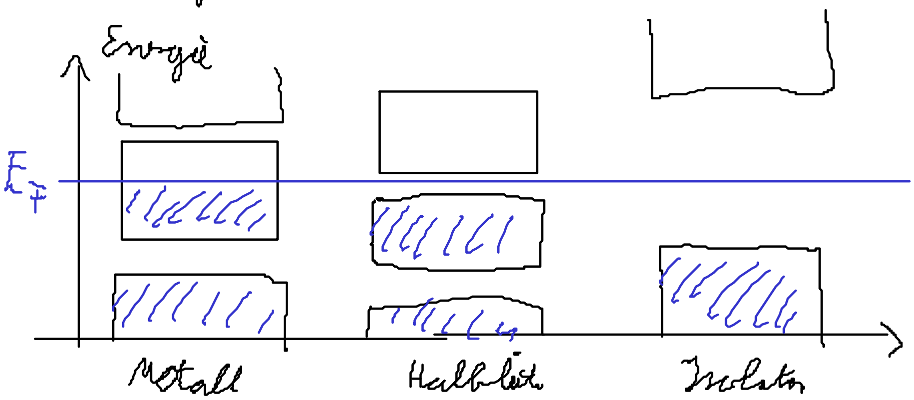
$$c_V^{spez} = c_V^{el} + c_V^{ph} = \gamma T + \begin{cases} \beta T^3 & T \ll \Theta_D \\ 3R & T \gg \Theta_D \end{cases}$$

Energiebänder

spez. Widerstand

„gute" Leiter $\rho \sim 10^{-10}\ \Omega\,\mathrm{cm}$

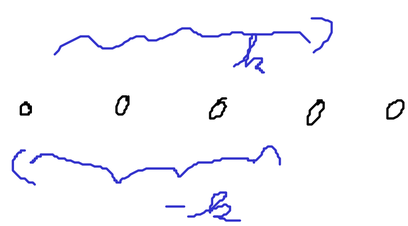
„gute" Isolatoren $\rho \sim 10^{22}\ \Omega\,\mathrm{cm}$



Modell der nahezu freien Elektronen

$$E_k = \frac{\hbar^2 k^2}{2m} \qquad \text{Wellenfkt.} \quad \psi_{\vec{k}}(\vec{r}) \sim e^{i\vec{k}\vec{r}}$$

Bragg-Reflektion



$$-\vec{k} + \vec{G} = \vec{k}'$$

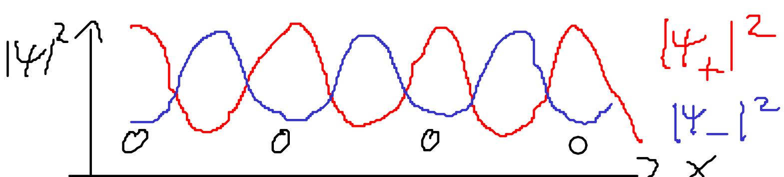
ein rez. Gittervektor

$$(-\vec{k} + \vec{G})^2 = \vec{k}^2 \Rightarrow 2kG = G^2 \Rightarrow k = \frac{G}{2} = \pm\frac{\pi}{a}n$$

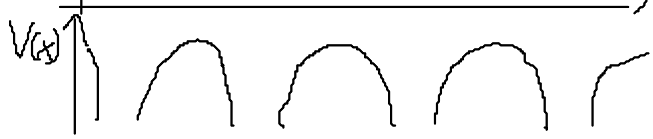
Zwei stehende Wellen möglich

$$\psi_+ \sim e^{i\frac{\pi x}{a}} + e^{-i\frac{\pi x}{a}} = 2\cos\frac{\pi x}{a}$$

$$\psi_- \sim e^{i\frac{\pi x}{a}} - e^{-i\frac{\pi x}{a}} = 2i\sin\frac{\pi x}{a}$$



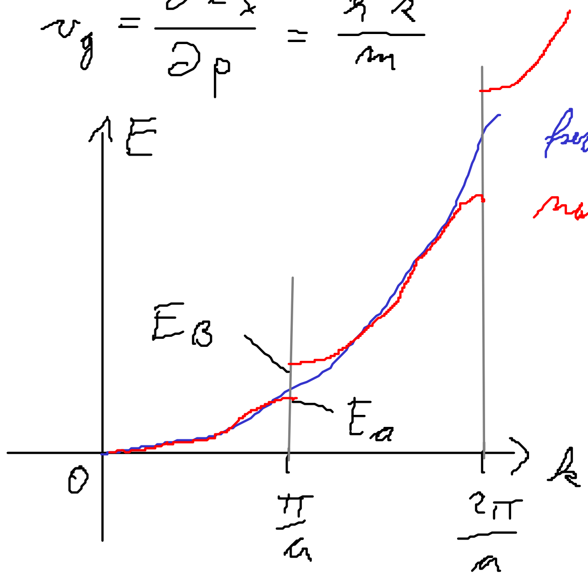
Dichte der El.



Potential

Gruppengeschwindigkeit $v_g = 0$ für $\psi_\pm$ (stehende Wellen)

$$v_g = \frac{\partial E_k}{\partial p} = \frac{\hbar k}{m} \qquad v_g\Big|_{k=\pm\frac{\pi}{a}} = 0$$



Erwartung $\quad E_+ < E_{frei} < E_-$

$$E_+ = E_a, \qquad E_- = E_B$$

Energielücke

$$U(x) = -U_0 \cos\frac{2\pi x}{a}$$

$$E_g = \frac{1}{a}\int dx\, U(x)\left[|\psi_-|^2 - |\psi_+|^2\right]$$

$$= -\frac{2U_0}{a}\int_0^a dx \cos\frac{2\pi x}{a}\left[1 - \cos\frac{2\pi x}{a} - 1 - \cos\frac{2\pi x}{a}\right]$$

$$= \frac{2U_0}{a}\int_0^a dx \cos^2\frac{2\pi x}{a} = \frac{U_0}{a}\left(x + \frac{a}{4\pi}\sin\frac{4\pi x}{a}\right)\Bigg|_0^a$$

$$= U_0$$