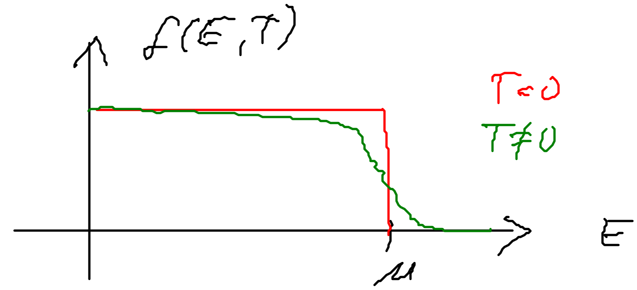
Sommerfeld-Theorie



$$f(E,T) = \frac{1}{e^{\frac{E-\mu}{kT}} + 1}$$

$$n = \frac{N}{V} = \int_0^\infty D(E)\, f(E,T)\, dE$$

- bei $T=0$: $\mu\big|_{T=0} = E_F$

$$n = \int_0^\infty D(E)\, f(E,0)\, dE = \frac{(2m)^{\frac{3}{2}}}{2\pi^2\hbar^3} \cdot \frac{2E_F^{\frac{3}{2}}}{3}$$

$$E_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{\frac{2}{3}}$$

- Fermi-Wellenvektor $k_F = (3\pi^2 n)^{\frac{1}{3}}$
- Fermi-Impuls $k_F \hbar$
- Fermi-Geschwindigkeit $v_F = \frac{\hbar}{m}(3\pi^2 n)^{\frac{1}{3}}$
- Fermi-Temperatur $T_F = \frac{E_F}{k_B}$

	$n\,[10^{28}\,m^{-3}]$	$k_F\,[Å^{-1}]$	$v_F\,[10^6\,\frac{m}{s}]$	$E_F\,[eV]$
Al	18,1	1,8	2,0	11,7
Cu	8,5	1,4	1,6	7,0
Ag	5,9	1,2	1,4	5,5

spezifische Wärme

$$C_V = \frac{\partial}{\partial T} u(T) = \frac{\partial}{\partial T}\int_0^\infty E\, D(E)\, f(E,T)\, dE = \frac{\partial}{\partial T}\frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}}\int_0^\infty \frac{E^{\frac{3}{2}}\, dE}{e^{\frac{E-\mu}{kT}}+1}$$

$u(T)$: innere Energie pro Volumen

$$u(T) \approx u(0) + \delta u(T)$$

$$u(0) = \int_0^{E_F} E\, D(E)\, dE \qquad f(E,0) = 1 \qquad D(E) = \frac{3}{2}\frac{n}{E_F}\left(\frac{E}{E_F}\right)^{\frac{1}{2}} = \frac{(2m)^{\frac{3}{2}}}{2\pi^2\hbar^3}\sqrt{E}$$

$$= \frac{3}{2}\frac{n}{E_F^{\frac{3}{2}}}\,\frac{2}{5}\,E_F^{\frac{5}{2}} = \frac{3}{5}\, n\, E_F = \frac{3}{5}\, n\, k_B T_F$$

$$\delta u(T) \approx n k_B T \underbrace{\frac{T}{T_F}}_{(*)} = \frac{n k_B}{T_F} T^2 \qquad C_V \approx \frac{\partial}{\partial T}\delta u(T) = \frac{2n k_B T}{T_F}$$

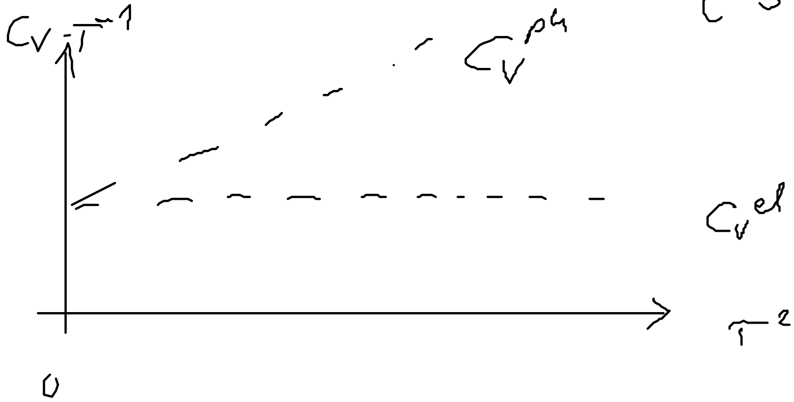
(*) Bruchteil der Elektr., die $k_B T$ aufnehmen

Näherung: $U(T) \approx U(0) + \frac{\pi^2}{6} D(E_F)(k_B T)^2$

$$\Rightarrow C_V = \frac{\pi^2}{3} \frac{T}{T_F} \frac{3 n k_B}{2} = \gamma T$$

- Sommerfeld-Konstante $\boxed{\gamma = \frac{\pi^2}{2} \frac{n k_B}{T_F}}$
- gesamte spez. Wärme

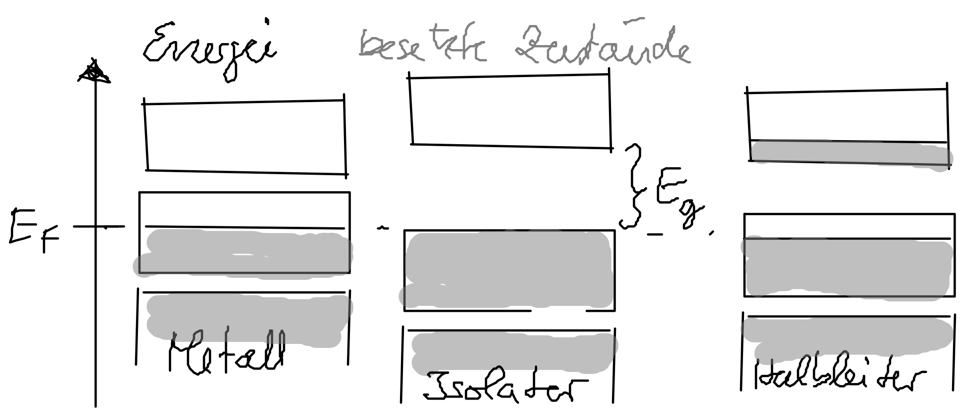
$$C_V^{ges} = C_V^{el} + C_V^{ph} = \gamma T + \begin{cases} \beta T^3 & T \ll \Theta_D \\ 3R & T \gg \Theta_D \end{cases}$$



Energiebänder

„gute" Leiter: $\rho \sim 10^{-10}\,\Omega m$

„gute" Isolatoren: $\rho \sim 10^{22}\,\Omega m$



Modell des nahezu freien Elektrons

$E_k = \frac{\hbar^2 k^2}{2m}$ $\quad \psi_k(\vec{r}) = e^{i\vec{k}\vec{r}}$

- Bragg-Reflektion: $-\vec{k} + \vec{G} = \vec{k}$

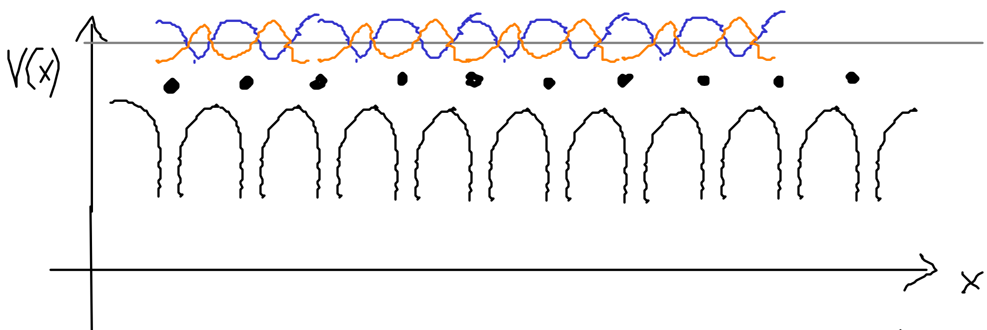
Streuvektor ↑ ↑ rezipr. Gittervektor

$(-\vec{k} + \vec{G})^2 = (\vec{k})^2 \qquad 2k\,G = G^2 \Rightarrow k = \frac{G}{2} = \pm \frac{\pi}{a} n$

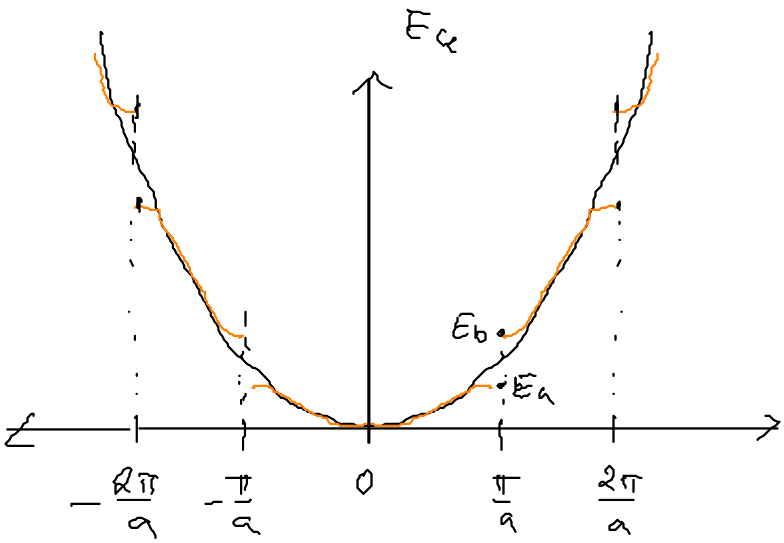
- zwei stehende Wellen

$$\psi_+ \propto e^{i\frac{\pi}{a}x} + e^{-i\frac{\pi}{a}x} = 2\cos\left(\frac{\pi x}{a}\right) \qquad |\psi_+|^2 \propto \cos^2\left(\frac{\pi x}{a}\right) = \frac{1}{2}\left(1+\cos\frac{2\pi x}{a}\right)$$

$$\psi_- \propto e^{i\frac{\pi}{a}x} - e^{-i\frac{\pi}{a}x} = 2i\sin\left(\frac{\pi x}{a}\right) \qquad |\psi_-|^2 \propto \frac{1}{2}\left(1-\cos\frac{2\pi x}{a}\right)$$



- Gruppengeschwindigkeit $v_g = \frac{\partial E_k}{\partial p} = \frac{\hbar k}{m}$ $\qquad v_g\Big|_{k=\pm\frac{\pi}{a}} = 0$



$E_+ < E_{frei} < E_-$

$E_a \qquad\qquad E_b$

- Energielücke E_g mit $V(x) \approx -V\cos\frac{2\pi x}{a}$

$$E_g = \frac{1}{a}\int dx\; V(x)\left(|\psi_-|^2 - |\psi_+|^2\right)$$

$$= -\frac{2V}{a}\int_0^a dx\cos\left(\frac{2\pi x}{a}\right)\left(1-\cos\left(\frac{2\pi x}{a}\right) - 1 - \cos\left(\frac{2\pi x}{a}\right)\right)$$

$$= \frac{2V}{a}\int_0^a dx\cos^2\left(\frac{2\pi x}{a}\right)$$

$$= \frac{2V}{a}\left(x + \frac{a}{4\pi}\sin\left(\frac{4\pi x}{a}\right)\right)\Big|_0^a = V$$