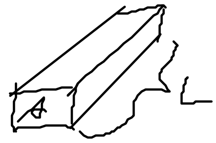
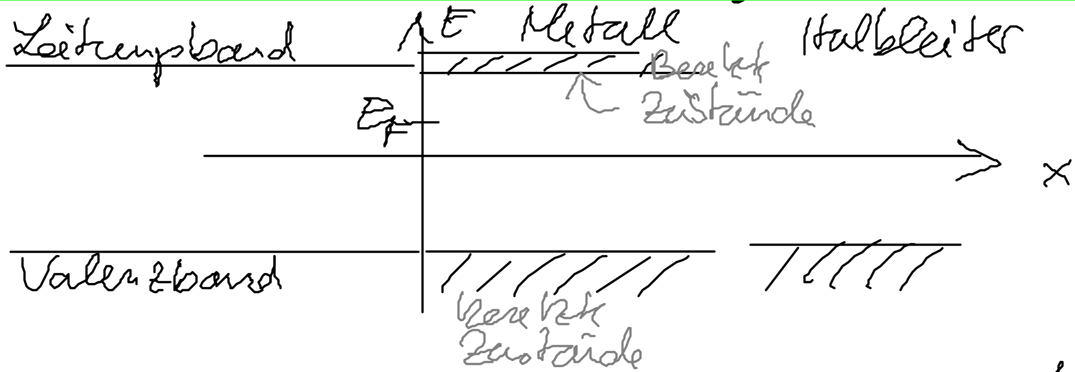
Übung Fabian Pauly

$L \to 0: g \to \infty$



Leitwert $g = \frac{\sigma A}{L}$

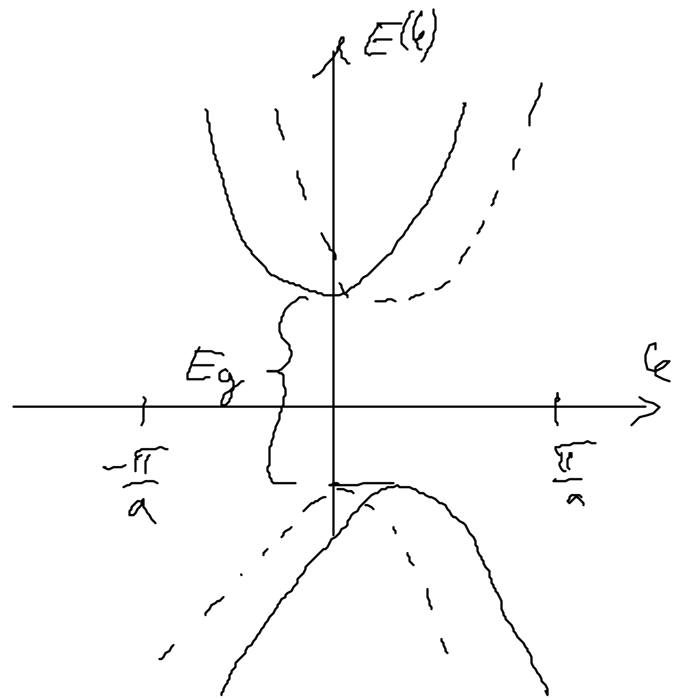
Modifikation der klassischen Physik in Nanostrukturen

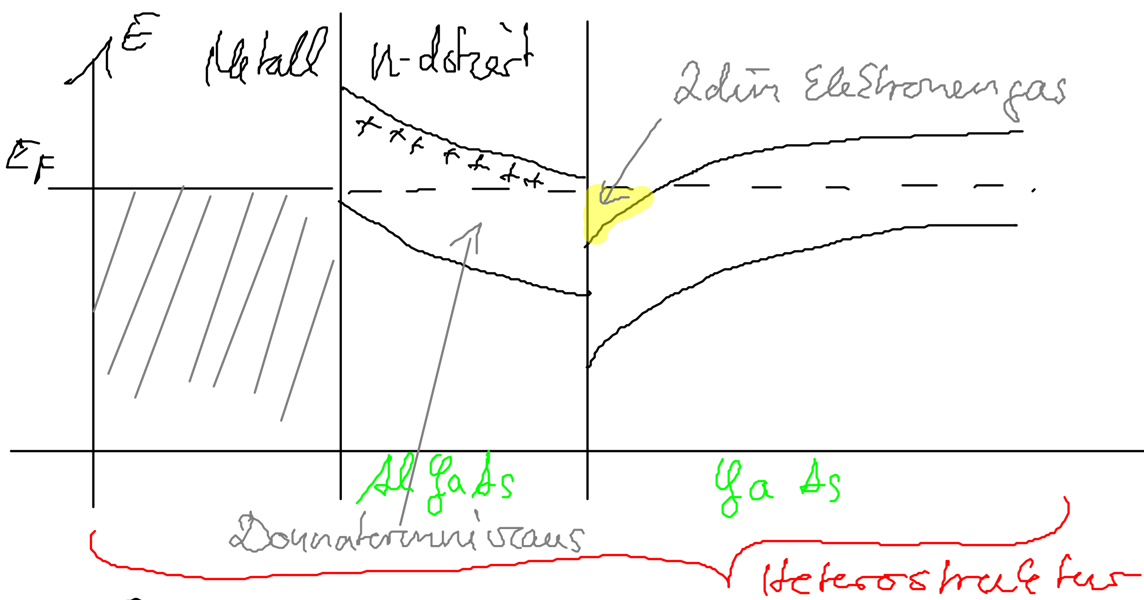


	GaAs	Si
$\frac{m}{m_e}$	0,067	0,19
λ_F	40 nm	100 nm
l_{imp}	$10^2 - 10^4$ nm	~100 nm
L_ϕ	200 nm $\sqrt{\frac{K}{T}}$	40–400 nm $\sqrt{\frac{K}{T}}$

L_ϕ Kohärenzlänge

	direkte Lücke	indirekte Lücke

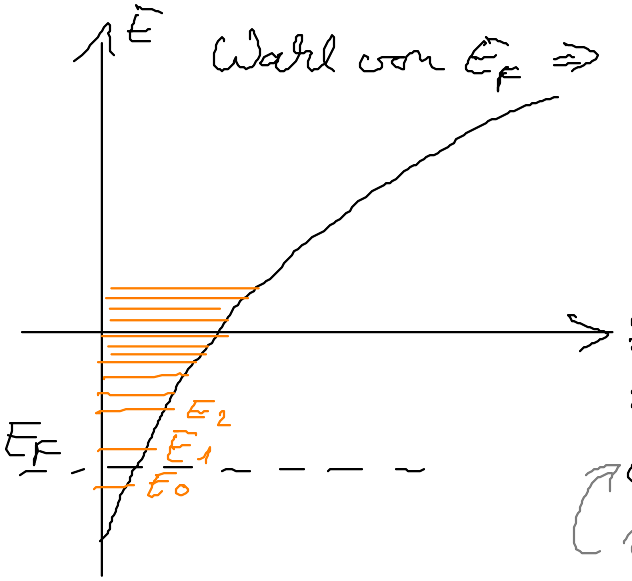




- Enger Raum (gelb) für Elektronen

$\Rightarrow$ Quantisierung der Energiezustände

Wahl von E_F ⇒ 2dim Problem



$$\Psi_n(\vec r) = \varphi_n(z)\exp[i(k_x\cdot x + k_y\cdot y)]$$

$$E(n,k_x,k_y) = -\frac{\hbar^2 (k_x^2+k_y^2)}{2m} + E_n$$

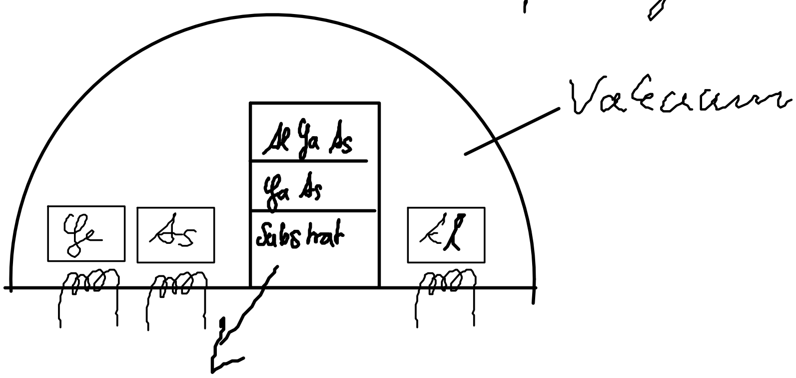
2-dim System: $E_0 \le E_F \le E_1$

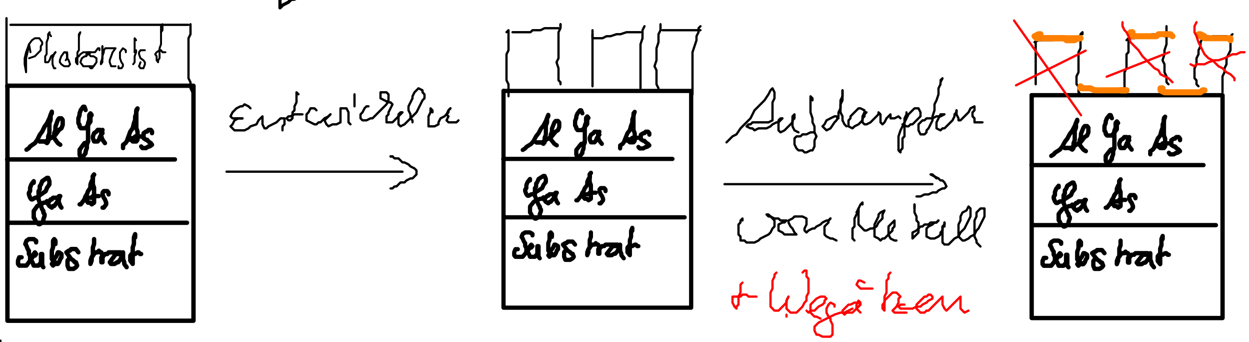
quasi 2dim Sys: $E_x \le E_F$ mit $x \ge 1$

Übergang zum 3-dim System

Herstellung eines Systems (Nanostruktur)

- MBE : Nuclear beam epitaxy





Andere Möglichkeit zur Herstellung von Nanostrukturen

- STM : Manipulation einzelner Atome („Scanning tunneling microscope")

1.3 Zustandsdichten

$\varepsilon_p = E_0 + \frac{p^2}{2m}$ (freie Elektronen) $d\varepsilon = \frac{p}{m}dp$ $\quad E_0 \ge 0$

Ebene Welle : $\Psi(x) = e^{ikx}$ Randbed: $\Psi(L) = e^{ikL} = \Psi(0)$

$\Rightarrow kL = 2\pi n \qquad k = \frac{2\pi n}{L} \Rightarrow \Delta k = \frac{2\pi}{L}$

3D: $\sum_{\vec p} g(\varepsilon_p) = V\cdot\int_{-\infty}^{\infty}\frac{d^3p}{(2\pi\hbar)^3}\, g(\varepsilon_p) = V\frac{4\pi}{(2\pi\hbar)^3}\int_{-\infty}^{\infty} dp\, p^2 g(\varepsilon_p)$

$= V\cdot\int_0^{\infty} d\varepsilon\, N^{3d}(\varepsilon)\, g(\varepsilon)$ g ist beliebig Funktion abh. von E

$$N^{3d}(\varepsilon) = \frac{m}{2\pi^2\hbar^3}\sqrt{2m(\varepsilon - E_0)}\,\Theta(\varepsilon - E_0)$$

$$N^{3d}(\varepsilon_F) = \frac{m}{2\pi^2\hbar^3}\,p_F\,;\hbar \quad , \quad n = \frac{4\pi}{3}p_F^3\frac{2}{(2\pi\hbar)^3}$$ Spin

2D: $$\sum_p g(\varepsilon_p) = A\cdot\int_{-\infty}^{\infty}\frac{d^2p}{(2\pi\hbar)^2}\,g(\varepsilon_p) = \frac{2\pi A}{(2\pi\hbar)^2}\int_0^{\infty}dp\,p\,g(\varepsilon_p)$$

$$= \frac{A}{2\pi\hbar^2}\int_{E_0}^{\infty}\frac{d\varepsilon\, m}{p}\,p\,g(\varepsilon) = A\int_0^{\infty}\underbrace{\frac{m}{2\pi\hbar^2}\Theta(\varepsilon - E_0)}_{N^{2d}(\varepsilon)}\,g(\varepsilon)$$

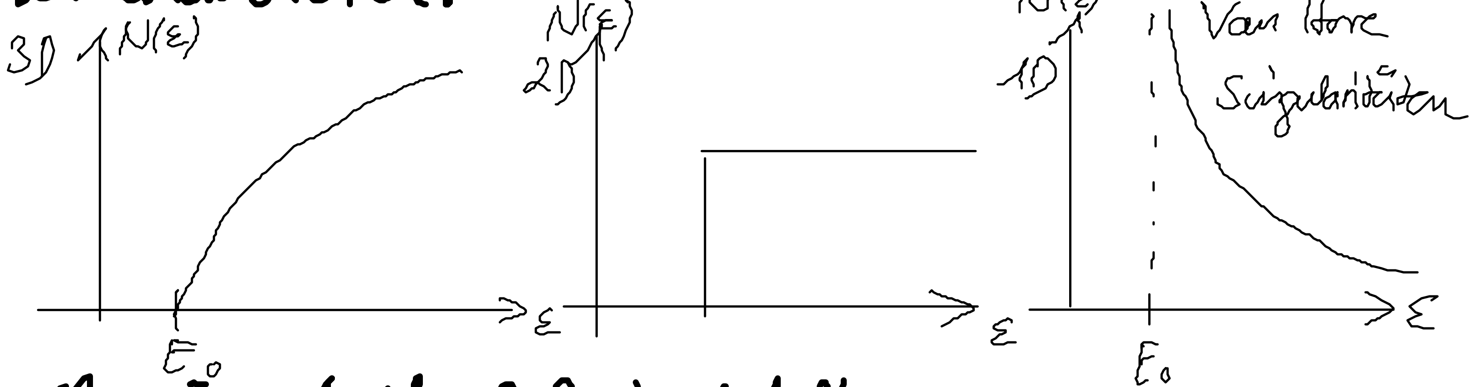
$$= A\int_0^{\infty}d\varepsilon\; N^{2d}(\varepsilon)\,g(\varepsilon)$$

$$N^{2d}(\varepsilon) = \frac{m}{2\pi\hbar^2}\cdot\Theta(\varepsilon - E_0) \quad , \quad n = 4\pi p_F^2\frac{2}{(2\pi\hbar)^2} = \frac{2}{\pi\hbar^2}p_F^2$$

1D: $$\sum_p g(\varepsilon_p) = L\cdot\int_{-\infty}^{\infty}\frac{dp}{2\pi\hbar}\,g(\varepsilon) = L\int_0^{\infty}d\varepsilon\,N^{1d}(\varepsilon)\,g(\varepsilon)$$

$$N^{1d}(\varepsilon) = \frac{2}{2\pi\hbar}\frac{1}{\frac{d\varepsilon}{dp}} = \frac{m}{\pi\hbar}\frac{1}{\sqrt{2m\varepsilon}}\,\Theta(\varepsilon - E_0)$$ von $\pm p$

Zustandsdichten:

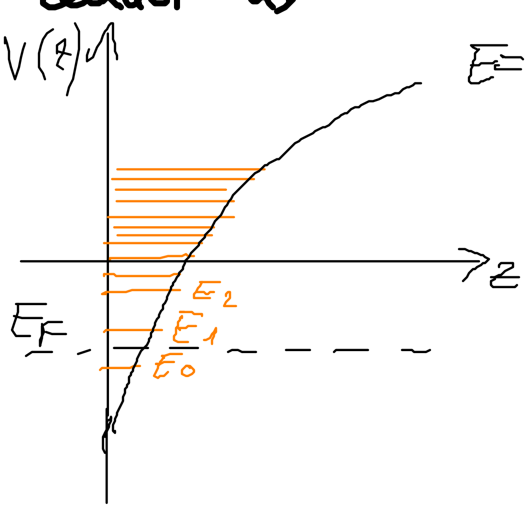


allgemeiner Ausdruck Zustandsdichte

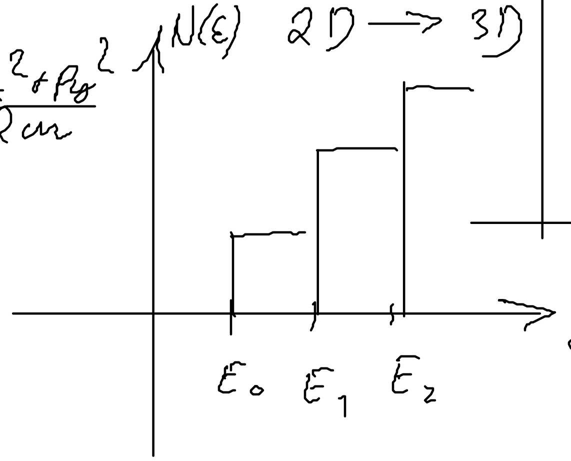
$$N(\varepsilon) = \sum_n\int\frac{d^dk}{(2\pi)^d}\,\delta(\varepsilon - \varepsilon_n(k))$$ d: # Dimensionen

pro Spin

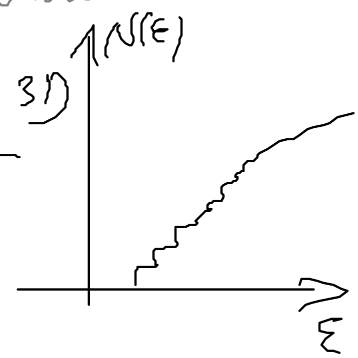
Quasi 2D



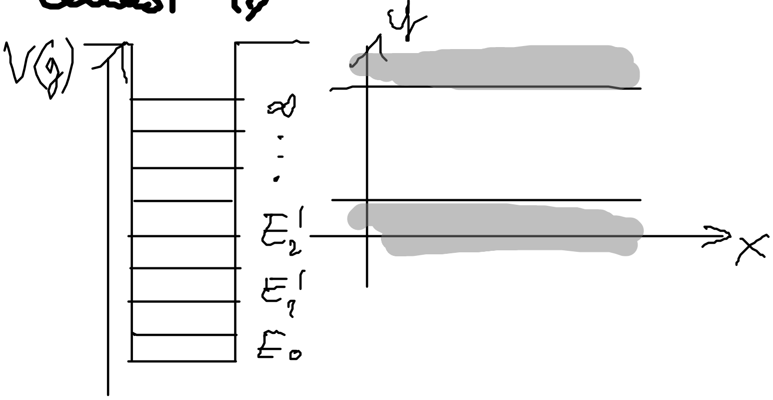
$$E = E_n + \frac{p_x^2 + p_y^2}{2m}$$



2D → 3D



Quasi 1D



$$\psi(\vec r) = \varphi_0(z)\,\chi_n(y)\,e^{ik_x x}$$

$$E = E_0 + E_n' + \frac{\hbar^2 k_x^2}{2m}$$

1.4 Leitfähigkeit

$\vec j = \sigma \vec E$ ort und zeitunabhängig

$\vec j(\vec q,\omega) = \hat\sigma(\vec q,\omega)\,\vec E(\vec q,\omega)$

Faltungstheorem: $\vec j(\omega) = \hat\sigma(\omega)\,\vec E(\omega)$

$$\vec j(t) = \int \frac{dt''}{\sqrt{2\pi}}\,\hat\sigma(t-t'')\,\vec E(t'')$$

$$\vec j(t) = \int \frac{d\omega}{\sqrt{2\pi}}\, f(\omega)\,e^{i\omega t}$$

$$f(\omega) = \int \frac{dt}{\sqrt{2\pi}}\, f(t)\,e^{-i\omega t}$$

1.4.1 Drude-Leitfähigkeit

$\vec j = n e \vec v$, $l = v \cdot \tau$ (l zwischen 2 Stößen)

$\dot{\vec p} = m\dot{\vec v} = e\vec E - \frac{m}{\tau}\vec v$ l: mittlere freie Weglänge

$\vec j = \frac{ne^2\tau}{m}\vec E$ n: Elektronendichte

$\sigma_0 = \frac{ne^2\tau}{m}$ (DC Gleichspannung) v: Elektronengeschwindigkeit

$\vec j(\omega) = \frac{ne^2}{m(\frac{1}{\tau}+i\omega)}\vec E(\omega)$ (Wechselsp.)

$\sigma(\omega) = \frac{ne^2\tau}{m(1+i\omega\tau)}$

1.5 Boltzmann

Verteilungsfkt. $f(\vec r,\vec p,t)$ μ: chemisches Potential

Gleichgewicht: $f(\varepsilon_p) = \frac{1}{\exp\left[\frac{\varepsilon_p-\mu}{k_B T}\right]+1}$

$\vec n(\vec r,t) \propto \sum_{p\sigma} f(\vec r,\vec p,t)$

$\vec j(\vec r,\vec p,t) \propto \sum_{p\sigma} \vec v\, f(\vec r,\vec p,t)$

$\vec v = \vec\nabla_p \varepsilon_p$ (Gruppengeschwindigkeit)

$$\frac{df(\vec r,\vec p,t)}{dt} = \frac{\partial f}{\partial t} + \frac{\partial \vec r}{\partial t}\vec\nabla_r f + \frac{\partial \vec p}{\partial t}\vec\nabla_p f = \frac{\partial f}{\partial t} + \frac{\vec p}{m}\vec\nabla_r f + \vec F\,\vec\nabla_p f$$

$$\boxed{\left(\frac{\partial}{\partial t} + \frac{\vec p}{m}\vec\nabla_r + e\vec E\,\vec\nabla_p\right) f = \left(\frac{df}{dt}\right)_{coll}}$$

← Kollisionsterm

Boltzmann-Gleichung

Trans- rein Streuung

Beispiel: Störstellenstreuung

← Geschw. an Fermi Kante

$$\left(\frac{\partial f}{\partial t}\right)_{imp} = -v_F\, n_{imp} \int d\Omega_{p'}\, \sigma(\Theta_{\vec p \vec p'}) \cdot \left(f(\vec p) - f(\vec p')\right)\Big|_{\varepsilon_{\vec p'} = \varepsilon_{\vec p}}$$

$$= -\frac{1}{\tau_{imp}} f(\vec p) + n_{imp}\, v_F \cdot \int d\Omega_{p'} \cdot \sigma(\Theta_{\vec p \vec p'})\, f(\vec p')\Big|_{\varepsilon_{\vec p} = \varepsilon_{\vec p'}}$$

$$(\tau_{imp})^{-1} = v_F \cdot \int d\Omega_{p'}\, \sigma(\vartheta_{pp'})$$

S-Wellenstreuung: $\sigma(\vartheta) = \text{const}$

$$\left(\frac{\partial f}{\partial t}\right) = -\frac{1}{\tau_{imp}}\left(f(\vec p) - \langle f(\vec p')\rangle\right), \quad \langle f(\vec p')\rangle = \int \frac{d\Omega_{\vec p'}}{4\pi} f(\vec p')$$

↳ Relaxationsapproximation