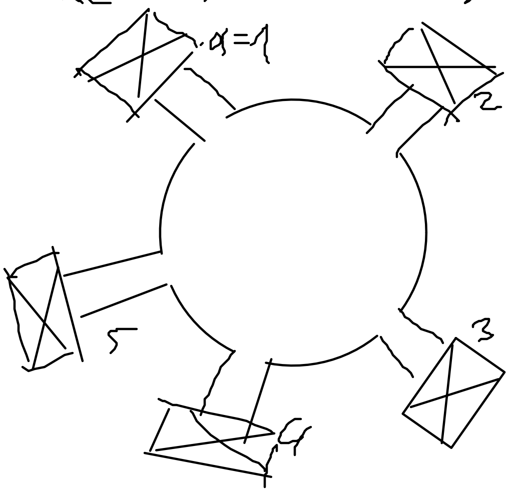
Landauer - Büttiker

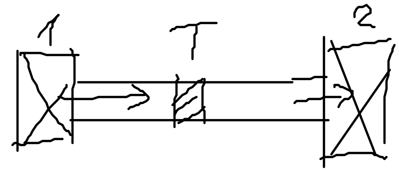
$$\frac{h}{2e} I_\alpha = [N_\alpha - R_\alpha]\,\mu_\alpha - \sum_{\beta\neq\alpha} T_{\alpha\beta}\,\mu_\beta$$

$$N_\alpha = \sum_{\beta\neq\alpha} T_{\beta\alpha} + R_\alpha$$



Einfache Beispiele

1) 2 Kontakt-System



$I_1 = I_2 = I$; $T_{12} = T_{21} = T$; $R_1 = R_2 = R = N - T$

$$\frac{h}{2e} I = (N-R)\,\mu_1 - T\mu_2 \qquad \text{I}$$

$$-\frac{h}{2e} I = (N-R)\,\mu_2 - T\mu_1 \qquad \text{II}$$

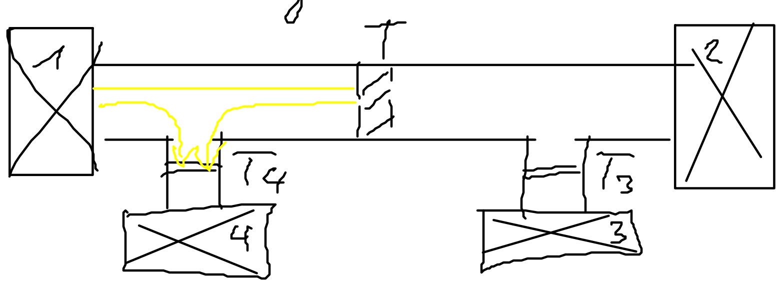
Differenz: $2\frac{h}{2e} I = (N-R)(\mu_1-\mu_2) + T(\mu_1-\mu_2)$

$$= (N-R+T)(\mu_1-\mu_2) = 2T(\mu_1-\mu_2)$$

$$\Rightarrow I = \frac{2e}{h}(\mu_1-\mu_2) = G_{2p}\,\frac{\mu_1-\mu_2}{e} \quad \text{mit} \quad G_{2p} = \frac{2e^2}{h}$$

Summe: $0 = (N-R)(\mu_1+\mu_2) - T(\mu_1+\mu_2) = 0$ ✓

2) 4-Kontakt-System mit 1 Kanal



Die Spannungskontakte sollen schwach angekoppelt sein: $T_3, T_4 \ll 1$

Zur Vereinfachung $T_3 = T_4 = \tau$, $R_3 = R_4$

$N_i = 1$

1. $\frac{h}{2e} I = (1-R_1)\mu_1 - T_{12}\mu_2 - T_{13}\mu_3 - T_{14}\mu_4$
2. $-\frac{h}{2e} I = (1-R_2)\mu_2 - T_{21}\mu_1 - T_{23}\mu_3 - T_{24}\mu_4$
3. $0 = (1-R_3)\mu_3 - T_{31}\mu_1 - T_{32}\mu_2 - \cancel{T_{34}\mu_4}$
4. $0 = (1-R_4)\mu_4 - T_{41}\mu_1 - T_{42}\mu_2 - \cancel{T_{43}\mu_3}$

Abschätzung $T_{12}=T_{21}=T+O(\tau)$

$$T_{13}=T\cdot\tau=T_{31}=T_{24}=T_{42}$$
$$T_{14}=\tau+R\cdot T=T_{41}=T_{23}=T_{32}$$
$$T_{34}=\tau^2 T=T_{43}\ll 1 \quad \text{wird vernachlässigt}$$

3 - 4: $0=(1-R_3)(\mu_3-\mu_4)-T_{31}(\mu_1-\mu_2)-T_{32}(\mu_2-\mu_1)$

$$0=(T_{13}+T_{23}+\cancel{T_{43}})(\mu_3-\mu_2)-(T_{32}-T_{31})(\mu_2-\mu_1)$$

$$\Rightarrow \mu_3-\mu_4=\frac{T_{32}-T_{31}}{T_{32}+T_{31}}(\mu_2-\mu_1)=\frac{\tau(1+R)-\tau T}{\tau(1+R)+\tau T}(\mu_2-\mu_1)$$
$$=\frac{2R}{2}(\mu_2-\mu_1)$$

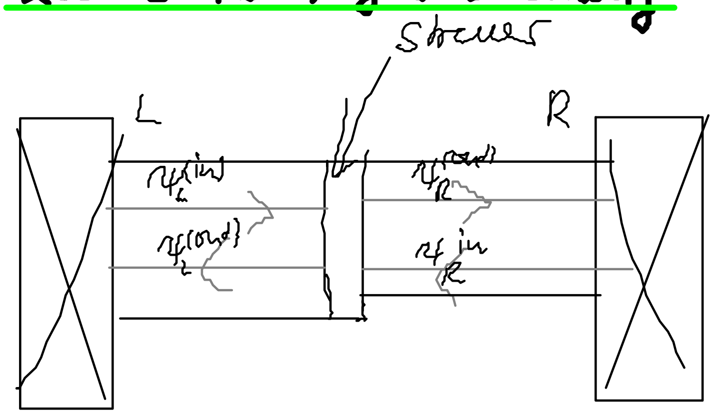
$$\boxed{\mu_3-\mu_4=R(\mu_2-\mu_1)}$$

1 - 2: $\frac{2\pi\hbar}{2e}I=(1-R)(\mu_1-\mu_2)-T_{12}(\mu_2-\mu_1)+O(\tau)$

$$=(1-R+T)(\mu_1-\mu_2)$$
$$=2T(\mu_1-\mu_2)$$

$$\Rightarrow \boxed{G_{2p}=\frac{eI}{\mu_1-\mu_2}=\frac{2e^2}{h}T}$$
$$\boxed{G_{4p}=\frac{eI}{\mu_4-\mu_3}=\frac{eI}{R(\mu_1-\mu_2)}=\frac{2e^2}{h}\frac{T}{R}}$$

2.8 S-Matrix für Streuung



$$\psi_L^{in}(x,y)=\sum_{n=1}^{N_L}a_n^L\,\chi_n^L(y)\,e^{ik_nx}$$
$$\psi_L^{out}(x,y)=\sum_{n=1}^{N_L}b_n^L\,\chi_n^L(y)\,e^{-ik_nx}$$
$$\psi_R^{in}(x,y)=\sum_{m=1}^{N_R}a_m^R\,\chi_m^R(y)\,e^{-ik_mx}$$
$$\psi_R^{out}(x,y)=\sum_{m=1}^{N_R}b_m^R\,\chi_m^R(y)\,e^{ik_mx}$$

- S ist eine quadratische $(N_L+N_R)^2$ Matrix
  $b=S\,a$ a und b sind (N_L+N_R)-Vektoren
  oder " $\psi^{out}=S\,\psi^{in}$ "
- Das ganze kann auf mehrere Kontakte verallgemeinert werden
  $S \Longleftrightarrow t_{\alpha\beta,mn}\,;\,r_{\alpha\alpha,mn}$

S-Matrix

$$\begin{pmatrix} b_1^L \\ b_2^L \\ \vdots \\ b_{N_L}^L \\ b_1^R \\ \vdots \\ b_{N_R}^R \end{pmatrix} = \left(\begin{array}{cccc|cccc} r_{11} & r_{12} & \cdots & r_{1N_L} & t'_{11} & \cdots & & t'_{1N_R} \\ \vdots & \ddots & & \vdots & \vdots & & & \\ r_{N_L1} & \cdots & & r_{N_LN_L} & t'_{N_L1} & \cdots & & t'_{N_LN_R} \\ \hline t_{11} & \cdots & & t_{1N_L} & r'_{11} & \cdots & & r'_{1N_R} \\ \vdots & & & & \vdots & & & \vdots \\ t_{N_R1} & \cdots & & t_{N_RN_L} & r'_{N_R1} & \cdots & & r'_{N_RN_R} \end{array}\right) \begin{pmatrix} a_1^L \\ a_2^L \\ \vdots \\ a_{N_L}^L \\ a_1^R \\ \vdots \\ a_{N_R}^R \end{pmatrix}$$

Eigenschaften

a) S ist unitär $\quad SS^\dagger = S^\dagger S = \mathbb{1}$

$$\Rightarrow \langle \psi^{in} | \psi^{in} \rangle = \int_{-\frac{L}{2}}^{\frac{L}{2}} dx\,dy \sum_{n=1}^{N_L+N_R} \sum_{n'=1}^{N_L+N_R} a_n^* a_{n'} \chi_n^*(y)\chi_{n'}(y) e^{\mp i k_n x} e^{\pm i k_{n'} x}$$

$$= L \sum_n |a_n|^2$$

$$\langle \psi^{out} | \psi^{out} \rangle = L \sum_n |b_n|^2 = L \sum_{n n' n''} S^*_{nn'} a^*_{n'} S_{nn''} a_{n''}$$

$$= L \sum_{nn'n''} S^\dagger_{n'n} S_{nn''} a^*_{n'} a_{n''}$$

mit $\psi^{in} = \begin{pmatrix} \psi_R^{in} \\ \psi_L^{in} \end{pmatrix}$

$$= L \sum_{nn''} \underbrace{(S^\dagger S)_{n'n''}}_{\delta_{n'n''}} a^*_{n'} a_{n''} = L \sum_n |a_n|^2$$

$$= \langle \psi^{in} | \psi^{in} \rangle$$

- Aus Unitarität von S folgt Norm, aber wir können dies nicht umdrehen und damit beweisen, dass S unitär sein muss.

Behauptung: S ist unitär folgt aus Stromerhaltung

$$I_L^{in} = \mathrm{Re}\left[\frac{e}{m} \int dy\, \psi_L^{in*} (-i\hbar \nabla_x) \psi_L^{in} \right]$$

$$= \frac{e\hbar}{m} \sum_{n=1}^{N_L} |a_n^L|^2 k_n$$

$$I_L = I_L^{in} - I_L^{out} = I_R = I_R^{out} - I_R^{in} \quad \text{(Stromerhaltung)} \quad \text{rechts = links}$$

$$\Rightarrow I^{in} = I_L^{in} + I_R^{in} = I_L^{out} + I_R^{out} = I^{out} \qquad in = out$$

$$\Rightarrow \sum_{n=1}^{N_L+N_R} |a_n|^2 k_n = \sum_{n=1}^{N_L+N_R} |b_n|^2 k_n$$

$$\frac{m}{e\hbar} I^{out} = \sum_{m=1} |b_m|^2 k_m = \sum_m \sum_{n n'} S^*_{mn} a^*_n S_{mn'} a_{n'} k_m$$

$$= \sum_{nn'} \sum_m S^\dagger_{nm} S_{mn'} k_m a^*_n a_{n'}$$

- Führe neue Matrix ein: $\tilde{S}_{mn} = S_{mn} \sqrt{\frac{k_m}{k_n}}$

$$S_{mn} = \sqrt{\frac{k_n}{k_m}} \tilde{S}_{mn}$$

$$\tilde{S}^\dagger_{mn} = \tilde{S}^*_{nm} = S^*_{nm} \sqrt{\frac{k_n}{k_m}} = S^\dagger_{mn} \sqrt{\frac{k_n}{k_m}}$$

$$S^\dagger_{mn} = \tilde{S}_{mn} \sqrt{\frac{k_m}{k_n}}$$

$$\frac{m}{e\hbar} I^{out} = \sum_{nn'} \sum_m \tilde{S}^\dagger_{nm} \sqrt{\frac{k_n}{\cancel{k_m}}} \tilde{S}_{mn'} \sqrt{\frac{k_{n'}}{\cancel{k_m}}} \cancel{k_m} a^*_n a_{n'}$$

$$= \sum_{nn'} \left(\sum_m \tilde{S}^\dagger_{nm} \tilde{S}_{mn'} \right) \sqrt{k_n k_{n'}} a^*_n a_{n'}$$

wenn $\tilde{S}^\dagger \tilde{S} = \mathbb{1}$ also $\tilde{S}$ unitär

$$= \sum_n |a_n|^2 k_n \quad \text{Stromerhaltung}$$

- Was bedeutet dies für S?

$$\sum_m S^\dagger_{nm} S_{mn'} = \sum_m \tilde{S}^\dagger_{nm} \sqrt{\frac{k_m}{k_n}} \sqrt{\frac{k_{n'}}{k_m}} \tilde{S}_{mn'}$$

$$= \sqrt{\frac{k_{n'}}{k_n}} (\tilde{S}^\dagger \tilde{S})_{nn'} = \mathbb{1}$$

b) Für Zeitumkehrinvariante Probleme $\Rightarrow S = S^T \Rightarrow t_{mn} = t_{nm}$

$$t_{\alpha\beta, mn} = t_{\beta\alpha nm}$$

$$G_{\alpha\beta} = G_{\beta\alpha}$$

Im Magnetfeld keine Zeitumkehrinvarianz

$$S(\vec{B}) = S^T(-\vec{B}) \Rightarrow t_{\alpha\beta mn}(\vec{B}) = t_{\beta\alpha nm}(-\vec{B})$$

$$\Rightarrow G_{\alpha\beta}(\vec{B}) \neq G_{\alpha\beta}(-\vec{B})$$