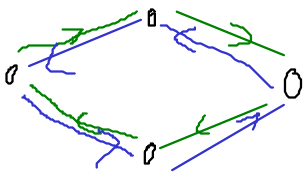
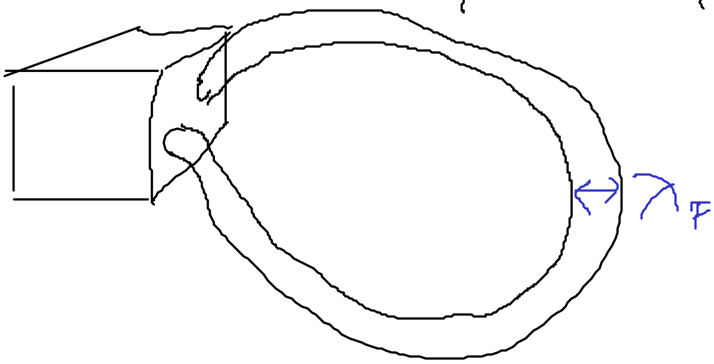
schwache Lokalisierung , $\ell_{imp} < L < L_\phi$



erhöhte Wahrscheinlichkeit für Rückkehr aufgrund der qm Interferenz zeitumgekehrter Wege

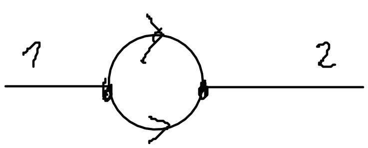


Rückkehrwahrscheinlichkeit für diffusives Problem berechnen

Zeiten kleiner τ_{imp} nicht betrachten, da Teilchen gestreut werden muss, bevor es zurückkehrt

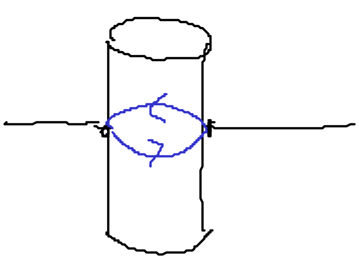
$$\Delta\sigma = -\frac{2e^2}{\pi\hbar} D \cdot \int_{\tau_{imp}}^{\infty} dt\, \tilde{W}_t$$

Vergleich zu Aharonov-Bohm-Effekt

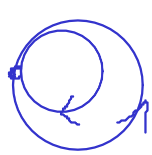


$$\Delta\Theta = 2\pi \frac{\phi}{\phi_0} \qquad \phi_0 = \frac{hc}{e}$$

Sharvin - Sharvin



$$\Delta\Theta = 2\pi \frac{2\phi}{\phi_0} = 2\pi \frac{\phi}{\tilde{\phi}_0}$$

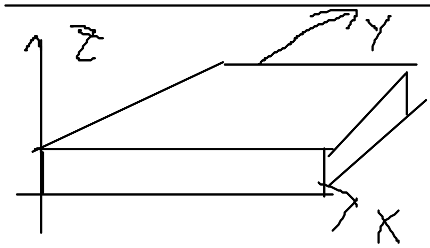


$$\tilde{\phi}_0 = \frac{hc}{2e}$$

Faktor 2 von zeitumgekehrten Wegen
(ähnlich Supraleitung $q = 2e$)

Feldstärke klein, d.h. 

(nicht  wie QHE)



schwache Lokalisierung im Magnetfeld (Film)

$$\left(\partial_t + D\left(-i\vec{\nabla} - \frac{2e}{\hbar c}A(\vec{r})\right)^2 + \frac{1}{\tau_\varphi}\right)\tilde{W}_t(\vec{r},0,t,0) = \delta(t)\,\delta(\vec{r})$$

(a) $\vec{B}$ parallel zum Film $\quad \vec{A} = \begin{pmatrix} 0 \\ B\left(z - \frac{a}{2}\right) \\ 0 \end{pmatrix} \quad \vec{B} = \vec{\nabla}\times\vec{A} = \begin{pmatrix} B \\ 0 \\ 0 \end{pmatrix}$

mittle über z

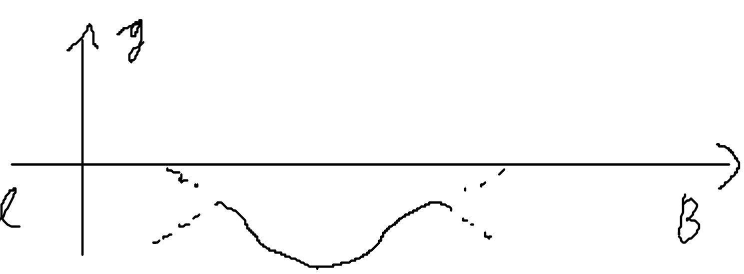
$$\left(\partial_t - D\left(\partial_x^2 + \partial_y^2\right) + \frac{1}{\tau_B} + \frac{1}{\tau_\varphi}\right)\tilde{W} = \frac{\delta(t)\,\delta(x)\,\delta(y)}{a}$$

$$\frac{1}{\tau_B} = D\left(\frac{2e}{\hbar c}\right)^2 \frac{1}{a}\int dz\, A^2(z) = \frac{1}{3}\cdot D\cdot\left(\frac{eBa}{\hbar c}\right)^2$$

Phasenkohärenz wird schneller zerstört

$$\frac{1}{\tau_\varphi} \longrightarrow \frac{1}{\tau_\varphi} + \frac{1}{\tau_B}$$

$$\Delta g = -\frac{e^2}{h}\ln\left(\frac{1/\tau_{imp}}{1/\tau_\varphi + 1/\tau_B}\right) \approx -\frac{e^2}{h}\left(\ln\left(\frac{\tau_\varphi}{\tau_{imp}}\right) - \tau_\varphi\frac{D}{3}\left(\frac{eBa}{\hbar c}\right)^2 + \dots\right)$$



anomaler Magnetwid.

(B) $\vec{B} \perp$ Film $\vec{A} = \begin{pmatrix} 0 \\ Bx \\ 0 \end{pmatrix}$ $\vec{B} = \begin{pmatrix} 0 \\ 0 \\ B \end{pmatrix}$

Diffusionsgl.

$$\partial_t + D\left(-\partial_x^2 + \left(-i\partial_y - \frac{2e}{\hbar c} Bx\right)^2 + \frac{1}{\tau_\varphi}\right)\tilde{W} = \delta(t)\,\frac{\delta(x)\,\delta(y)}{a}$$

$$\tilde{W} = \frac{1}{a}\sum_n \int \frac{dk}{2\pi}\, \psi_n\left(x_f - \frac{\hbar c k}{2eB}\right)\psi_n\left(x_i - \frac{\hbar c k}{2eB}\right)$$

$$\cdot \exp\left(ik(y_f - y_i) - \omega_n t_0 - \frac{t_0}{\tau_\varphi}\right)$$

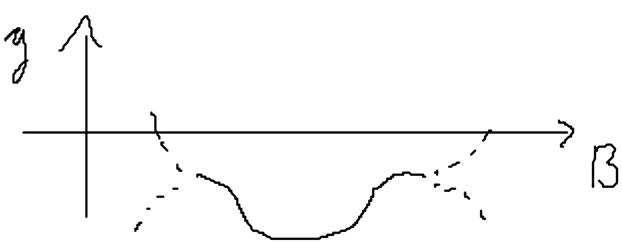
$$\omega_n = \frac{4eB}{\hbar c}\left(n + \frac{1}{2}\right)$$

ψ_n sind Eig.Fkt. von $D\left(-\partial_x^2 + \left(\frac{2eB}{\hbar c}\right)^2 x^2\right)$

. . . (Integration) . . .

$$\Delta g = -\frac{2e^2}{\pi\hbar}\int_{\tau_{imp}}^{\infty} dt\, \tilde{W}_t$$

$$\underset{B\to 0}{=} -\frac{e^2}{2\pi^2\hbar}\left(\ln\left(\frac{\tau_\varphi}{\tau_{imp}}\right) - \frac{1}{6}\left(\frac{eBD\tau_\varphi}{\hbar c}\right)^2 + \dots\right)$$



Erhöhung von g durch B macht Δg beobachtbar

5.2.1 Spin-flip und Spin-Bahn-Streuung

$$f(p, p') = u_{sf}\,\vec{\sigma}\,\vec{J} + i\,u_{so}\,\vec{\sigma}\,[\vec{p} \times \vec{p}\,']$$

Spin e^- . . . an Spin impurity

Spin-Bahn

$$\sigma, p \longrightarrow \times \longrightarrow \sigma', p'$$

$$\frac{1}{\tau_{sf}} = \pi N(E_F)\, N_S\, |U_{sf}|^2\, S(S+1)$$

Konzentration der Verunreinigungsspins

$$\frac{1}{\tau_{so}} = \pi N(E_F)\, N_i\, |U_{so}|^2 \langle [p \times p'] \rangle$$

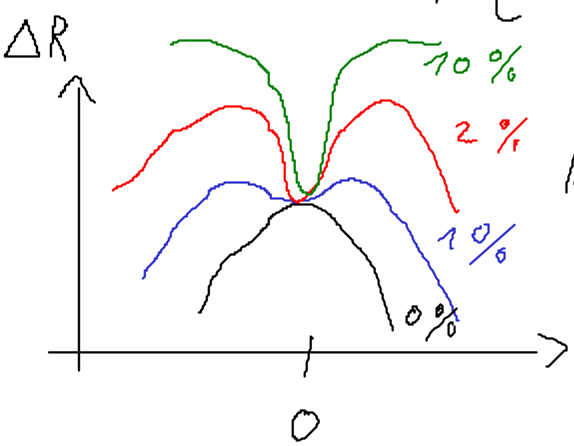
Konz. der Störstellen

$$\frac{1}{\tau_{sg}} = \frac{2}{\tau_{sf}} + \frac{1}{\tau_\varphi} \qquad \frac{1}{\tau_{tp}} = \frac{4}{3\tau_{sf}} + \frac{2}{3\tau_{sf}} + \frac{1}{\tau_\varphi}$$

singlett   triplett

$$\Rightarrow \Delta g = -\frac{e^2}{2\pi^2\hbar}\left\{ \ln\left(\frac{\tau_{tp}}{\tau_{sg}\,\tau_{imp}^2}\right) - \frac{1}{6}\left(\frac{eBD}{\hbar c}\right)^2 \left(3\tau_{tp}^2 - \tau_{sg}^2\right) + \dots \right\}$$

je nach Aufteilung kann Korrektur positiv oder negativ sein

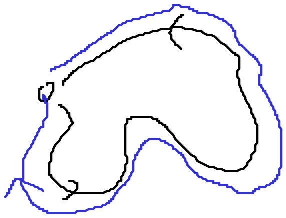


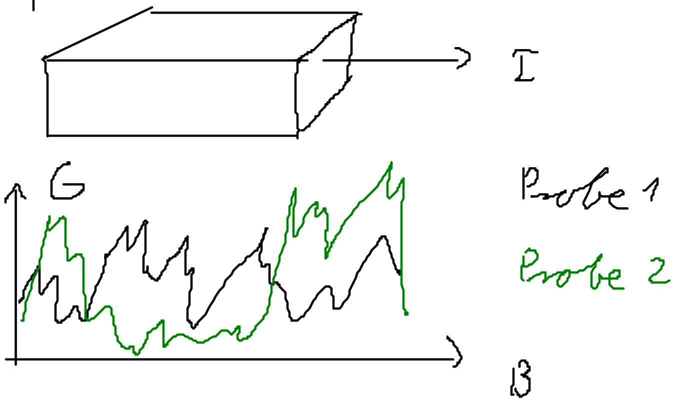
Konz. Gold (Au)

Referenz: S. Chakravarty und A. Schmid
Phys. Rep. 140, 193 (1986)
G. Bergmann, Phys. Rep. 107, 1 (1984)

5.2.2 Probenspezifische Fluktuationen

$l_{imp} < L < L_\varphi$





G weist charakteristische, reproduzierbare Fluktuationen auf als Fkt. von B.

(kein Rauschen sondern charakterist. für Störstellen-konfiguration)

5.2.3 Universal conductance fluctuations (UCF)

Universelle Leitwertfluktuationen

– Metallischer Fall

$$G = \frac{2e^2}{h} \sum_{\alpha,\beta=1}^{N} |t_{\alpha\beta}|^2$$

$\boxed{2D}$

$$G_{classic} = \sigma \frac{W}{L}$$

$\left(\frac{\text{Breite}}{\text{Länge}}\right)$

$$\sigma = \frac{n^{2D} e^2 \tau}{m}$$

Drude

$$n^{2D} = 2 \frac{\pi k_F^2}{(2\pi)^2}$$

$$\Rightarrow \sigma = \frac{e^2}{h} k_F l_{imp}$$

$v_F \cdot \tau_{imp}$

$$N = \frac{W k_F}{\pi}$$

Kanäle

$$G = \frac{2e^2}{h} \cdot \frac{\pi l_{imp}}{2L} N$$

$$\sum_{\alpha\beta} \langle |t_{\alpha\beta}|^2 \rangle = \frac{\pi l_{imp}}{2L} N$$

$$\sum_{\alpha,\beta=1}^{N} |t_{\alpha\beta}|^2 = N - \sum_{\alpha,\beta=1}^{N} |r_{\alpha\beta}|^2$$

Annahme: Rückstreuwahrscheinlichkeiten unkorreliert

$$\sum_{\alpha,\beta} \langle |r_{\alpha\beta}|^2 \rangle = N^2 \langle |r_{\alpha\beta}|^2 \rangle$$

$\langle \cdots \rangle \hat{=}$ Impurity average ; Störstellenmittelung

$$\langle |r_{\alpha\beta}|^2 \rangle = \frac{1}{N} \left(1 - \frac{\pi\, l_{imp}}{2L} \right)$$

$$\langle \delta G^2 \rangle = \langle (G - \langle G \rangle)^2 \rangle$$

$$\langle G^2 \rangle = \frac{2e^2}{h} \langle \sum_{\alpha\beta\gamma\delta} |t_{\alpha\beta}|^2 \cdot |t_{\gamma\delta}|^2 \rangle$$

$$\langle \delta G^2 \rangle = \left(\frac{2e^2}{h} \right)$$

$$\left[\langle (N - \sum_{\alpha\beta} |r_{\alpha\beta}|^2)^2 \rangle - (N - \sum_{\alpha\beta} \langle |r_{\alpha\beta}|^2 \rangle)^2 \right]$$

? $\langle N^2 \rangle - N^2 = 0$?

$$= \left(\frac{2e^2}{h}\right)^2 \left[\langle (\sum_{\alpha\beta} |r_{\alpha\beta}|^2)^2 \rangle - \langle \sum_{\alpha\beta} |r_{\alpha\beta}|^2 \rangle^2 \right]$$

$$= \left(\frac{2e^2}{h}\right)^2 \langle \delta (\sum_{\alpha\beta} |r_{\alpha\beta}|^2)^2 \rangle$$

$$= \left(\frac{2e^2}{h}\right)^2 N^2 \langle \delta (|r_{\alpha\beta}|^2)^2 \rangle$$

$$\langle \delta (|r_{\alpha\beta}|^2)^2 \rangle = \langle |r_{\alpha\beta}|^4 \rangle - \langle |r_{\alpha\beta}|^2 \rangle^2$$

$$|r_{\alpha\beta}|^2 = \sum_{\bar{i}\bar{j}} A^{*\,\bar{i}}_{\alpha\beta} A^{\bar{j}}_{\alpha\beta}$$

$$\langle |r_{\alpha\beta}|^2 \rangle = \sum_{\bar{i}} |A^{\bar{i}}_{\alpha\beta}|^2$$

$$(\langle r_{\alpha\beta}\rangle^4) = \langle \sum_{ijkl} A^{i*} A^{j} A^{k*} A^{l} \rangle$$

$$= \sum_{ijkl} |A^{j}|^2 \, |A^{k}|^2 \, (\delta_{ij}\,\delta_{kl} + \delta_{il}\,\delta_{kj})$$

$$= 2 \langle |r_{\alpha\beta}|^2 \rangle$$

$$\langle \delta G^2 \rangle = \left(\frac{2e^2}{h}\right)^2 N^2 \langle |r_{\alpha\beta}|^2 \rangle$$ (Annahme war $L \gg l_{mfp}$)

$$\propto \left(\frac{2e^2}{h}\right)^2 \underbrace{c}_{O(1)}$$ universelle Leitwertfluktuation

δG unabhängig von Dimension $D = 1, 2, 3$
und $\langle G \rangle$

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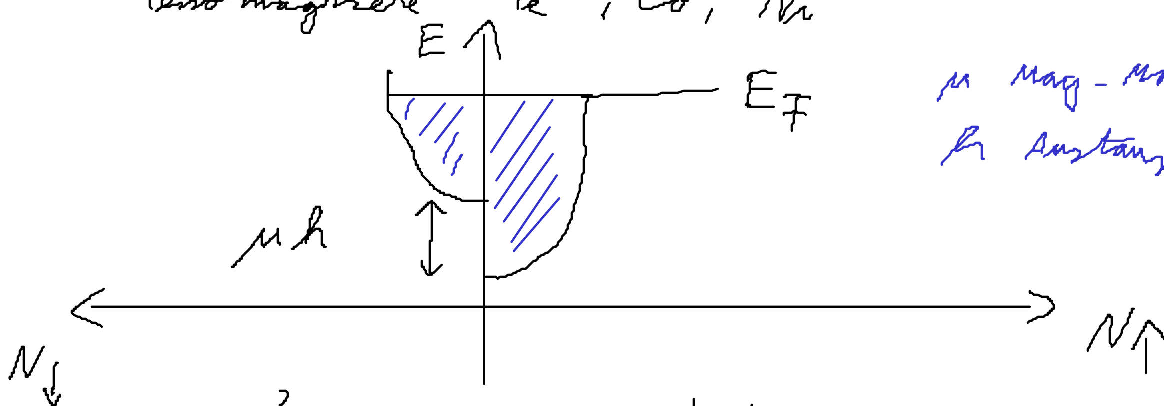
6 Spin-Elektronik (Spintronic)

Spineffekte: Magnetismus

Ferromagnete Fe, Co, Ni



μ Mag-Moment
h Austauschfeld

$$H = \frac{p^2}{2m} + U(x) + \underbrace{\mu}_{\text{mag. Moment}} \underbrace{\vec{h}}_{\text{Austauschfeld}} \vec{S}$$