# Maxwell-Boltzmann-Gas

- kanonisch

$$\int \frac{dp_x}{2\pi\hbar} \exp\left[-\beta \frac{p_x^2}{2m}\right] = \sqrt{\frac{2m\pi}{\beta}} \frac{1}{2\pi\hbar} \qquad \alpha = \frac{\beta}{2m}$$

$$S = -\frac{\partial F}{\partial T}\Big|_{N,V} = -\frac{F}{T} \qquad F = U - TS \qquad \frac{\partial F}{\partial T} = -a \frac{1}{\cancel{b}T^{\frac{3}{2}}} \frac{3}{2} T^{\frac{1}{2}} \cancel{b}$$

$$= a \ln(b T^{\frac{3}{2}})$$

- großkanonisch

$$S = \left(\frac{\partial \Omega}{\partial T}\right)_{\mu,V} = -\frac{\Omega}{T} + k_B T \mu \exp[\beta\mu] Z_1 \left(\frac{\partial \beta}{\partial T}\right) + k_B T \exp[\beta\mu] \frac{\partial Z_1}{\partial T}$$

$$\boxed{\begin{aligned} \frac{\partial \beta}{\partial T} &= \frac{\partial}{\partial T}\left(\frac{1}{k_B T}\right) = -\frac{1}{k_B T^2} = -\frac{\beta}{T} \\ Z_1 &= \frac{V}{\lambda_T^3} = a T^{\frac{3}{2}} \\ \frac{\partial Z_1}{\partial T} &= \frac{3}{2} a T^{\frac{1}{2}} = \frac{3}{2} \frac{Z_1}{T} \end{aligned}}$$

$$\Omega = F - \mu N = U - TS - \mu N \qquad U = \Omega + TS + \mu N$$

## Quantenzustände

$$\hat{h} = \frac{\hat{p}^2}{2m} \qquad \varphi_p = \frac{1}{\sqrt{V}} \exp\left[i \frac{\vec{p}\vec{r}}{\hbar}\right] \qquad \varepsilon_p = \frac{p^2}{2m}$$

$$\prod_\lambda = \exp[-\beta(\varepsilon_\lambda - \mu)] = \exp\left[\sum_\lambda \exp[-\beta(\varepsilon_\lambda - \mu)]\right] = \exp\left[e^{\beta\mu} \sum e^{-\beta\varepsilon_\lambda}\right]$$

## Warum versagt Maxwell-Boltzmann? Besetzungszahl $n_\lambda$

$\varphi_\lambda(x_1)\, \varphi_\lambda(x_2)\, \varphi_\lambda(x_3)$ nicht betrachtet: $n_\lambda = 3$ Zustand 3-mal besetzt

- überpermutiert
- Korrektur durch weglassen von Faktor $\frac{1}{N!}$

## ideales Bose-Gas

$\Psi$ muss symmetrisch sein, $n_\lambda = 0, 1, 2, \ldots \infty$

Sym. Reihe $\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$ $\qquad \tilde{\varepsilon} = \varepsilon_\lambda - \mu$

$$W_\lambda(n_\lambda) = \frac{1}{Z_\lambda} \exp[-\beta(\varepsilon_\lambda - \mu) n_\lambda] = (1 - \exp[-\beta\tilde{\varepsilon}]) \exp[-\beta\tilde{\varepsilon} n_\lambda]$$

$$\langle n_\lambda \rangle = \sum_{n_\lambda} W(n_\lambda)\, n_\lambda = \left(\sum_{n_\lambda} \exp[-\beta\tilde{\varepsilon} n_\lambda]\, n_\lambda\right)(1 - \exp[-\beta\tilde{\varepsilon}])$$

$$= \left(-\frac{\partial}{\partial(\beta\tilde{\varepsilon})} \sum \exp[-\beta\tilde{\varepsilon} n_\lambda]\right)(1 - \exp[-\beta\tilde{\varepsilon}])$$

$$= \left(-\frac{\partial}{\partial(\beta\tilde{\varepsilon})} Z_\lambda\right) \frac{1}{Z_\lambda}$$

$$Z_\lambda = \frac{1}{1 - \exp[-\beta\tilde{\varepsilon}]} \quad ; \quad \frac{\partial}{\partial(\beta\tilde{\varepsilon})} = -\frac{1}{1 - \exp[-\beta\tilde{\varepsilon}]}\left(-\exp[-\beta\tilde{\varepsilon}]\right)$$

$\Rightarrow$ **Bose Funktion** $W_\lambda(n_\lambda)$