Das ideale Bose-Gas

$$\{n_\lambda\} = \{n_1, n_2, \dots\}$$

$$Z_G = \sum_{\{n_\lambda\}} \exp\left[-\beta \sum_\lambda n_\lambda (\varepsilon_\lambda - \mu)\right] = \prod_\lambda z_\lambda$$

$$W_{\{n_\lambda\}} = \frac{1}{Z_G} \exp\left[-\beta \sum_\lambda n_\lambda (\varepsilon_\lambda - \mu)\right]$$

$$W_{\lambda_1}(n_{\lambda_1}) = \sum_{\{n_\lambda, \lambda \neq \lambda_1\}} \frac{1}{\prod_\lambda z_\lambda} \prod_{\lambda \neq \lambda_1} z_\lambda \exp\left[-\beta n_{\lambda_1}(\varepsilon_{\lambda_1} - \mu)\right]$$

$$= \frac{1}{z_{\lambda_1}} \exp\left[-\beta n_{\lambda_1}(\varepsilon_{\lambda_1} - \mu)\right]$$

$$\langle n_\lambda \rangle = \frac{1}{z_\lambda} \sum_{n_\lambda} n_\lambda \exp[-n_\lambda x] \qquad \boxed{\text{mit } \beta(\varepsilon_\lambda - \mu) = x}$$

$$\boxed{z_\lambda = \frac{1}{1 - e^{-x}}}$$

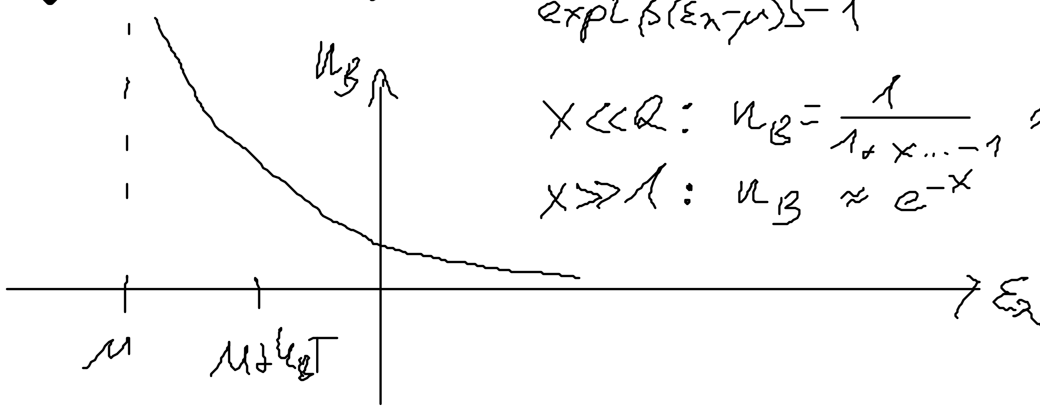
$$\langle n_\lambda \rangle = (1 - e^{-x})(-1)\frac{\partial}{\partial x} \sum e^{-n_\lambda x} = (e^{-x} - 1)\frac{\partial}{\partial x}\left(\frac{1}{1 - e^{-x}}\right)$$

$$= (e^{-x} - 1) \cdot \frac{-e^{-x}}{(1 - e^{-x})^2} = \frac{e^{-x}}{1 - e^{-x}} = \frac{1}{e^x - 1}$$

Bosefunktion: $n_B(\varepsilon_\lambda) = \dfrac{1}{\exp[\beta(\varepsilon_\lambda - \mu)] - 1}$

$x \ll 1: \quad n_B = \dfrac{1}{1 + x \dots - 1} \approx \dfrac{1}{x}$

$x \gg 1: \quad n_B \approx e^{-x}$



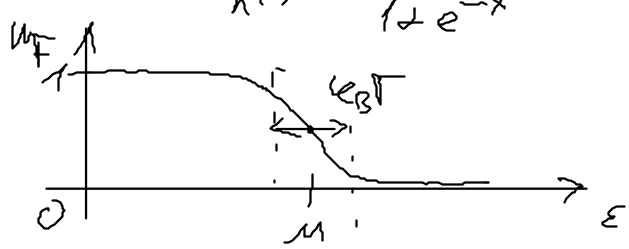
Das ideale Fermi-Gas

$$z_\lambda = 1 + e^{-x} \qquad W_\lambda(n_\lambda) = \frac{1}{z_\lambda} e^{-n_\lambda x} \qquad W_\lambda(0) = \frac{1}{1 + e^{-x}}$$

$$\langle n_\lambda \rangle = \frac{1}{e^x + 1} \qquad W_\lambda(1) = \frac{e^x}{1 + e^{-x}}$$

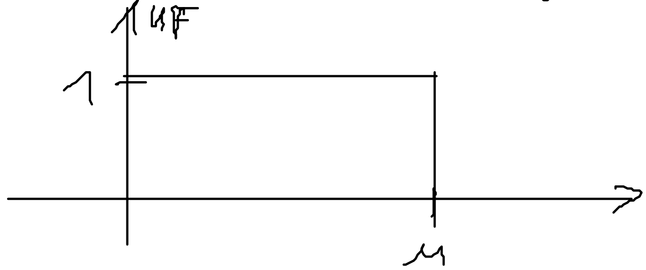
Fermifunktion

$$n_F(\varepsilon) = \frac{1}{\exp[\beta(\varepsilon - \mu)] + 1}$$



$x \ll 1$: $(\varepsilon-\mu)\beta \ll -1$, $\varepsilon-\mu \ll -\frac{1}{\beta} = -k_B T$

M. Boltzmann ist eine Näherung für große Temperaturen.



$T = 0$

$n_F = \Theta(\mu - \varepsilon)$

$$\langle N \rangle = -\frac{\partial \Omega}{\partial \mu} = -\frac{\partial}{\partial \mu}\left(-k_B T\right) \sum_n \ln\left(1+e^{-x}\right)$$

$$= \cancel{k_B T} \sum_n \frac{\cancel{\beta} e^{-x}}{1+e^{-x}} = \sum_n \langle n_n \rangle$$

$$S = -\frac{\partial \Omega}{\partial T} = \frac{\partial}{\partial T}\left(k_B T \sum_n \ln\left(1+e^{-x}\right)\right)$$

$$= k_B \sum_n \ln\left(1+e^{-x}\right) + k_B \sum_n \frac{\beta(\varepsilon_n-\mu)}{1+e^{-x}} e^{-x}$$

$$= k_B \sum_n \ln\left(1+e^{-x}\right) + \frac{x}{1+e^{-x}} e^{-x}$$

$$n_F(\varepsilon_n) = \frac{1}{e^x+1} \qquad 1-n_F = 1-\frac{1}{e^x+1} = \frac{e^x}{e^x+1} = \frac{1}{e^{-x}+1}$$

$$x = \ln e^x$$

$$e^x = \frac{1}{n_F} - 1 = \frac{1-n_F}{n_F}$$

$$S = k_B \sum_n \left(-\ln\left(1-\langle n \rangle\right) + \left(1-\langle n \rangle\right) \ln\left(\frac{1-n}{n}\right)\right)$$