Ideales Bose Gas

Riemannsche Zeta Fkt: $\zeta(x) = \sum_{n=1}^{\infty} \frac{1}{n^x}$

$U = \langle E \rangle = \langle \sum_\lambda \varepsilon_\lambda n_\lambda \rangle$

$z \ll 1 \qquad e^{\beta\mu} \ll 1 \qquad \mu \ll -k_B T$

$$n = (2s+1) \frac{1}{\lambda_T^3} \left(z + \frac{z^2}{2^{3/2}} \right) \qquad e^{\beta\mu} = z \approx \frac{\lambda_T^3 n}{(2s+1)}$$

$$\Rightarrow \mu = \frac{1}{\beta} \ln \left(\frac{\lambda_T^3 n}{2s+1} \right)$$

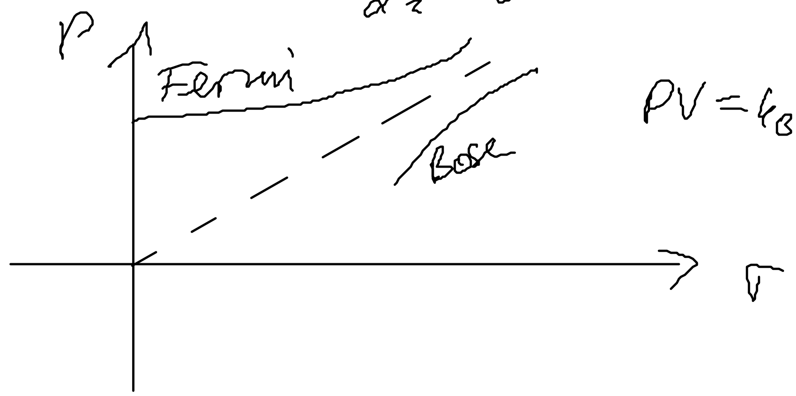
$$\frac{n \lambda_T^3}{2s+1} = z + \left(\frac{\lambda_T^3 n}{2s+1} \right)^2 \frac{1}{2^{3/2}}$$

$$\Rightarrow z = \frac{n \lambda_T^3}{2s+1} \left(1 - \frac{n \lambda_T^3}{2s+1} \frac{1}{2^{3/2}} \right)$$

$$\Omega = -k_B T (2s+1) \frac{V}{\lambda_T^3} \left(\frac{n \lambda_T^3}{2s+1} \left(1 - \frac{n \lambda_T^3}{2s+1} \frac{1}{2^{3/2}} \right) + \left(\frac{n \lambda_T^3}{2s+1} \right)^2 \frac{1}{2^{5/2}} \right)$$

$$\frac{1}{2^{3/2}} - \frac{1}{2^{5/2}} = \frac{1}{2^{3/2}} \left(1 - \frac{1}{2} \right) = \frac{1}{2^{5/2}}$$

$$\Rightarrow PV = k_B T N \left(1 - \frac{1}{2^{5/2}} \frac{n \lambda_T^3}{2s+1} \right)$$



$PV = k_B N T$

Bose - Einstein - Condensation

$$\frac{z}{1-z} = \frac{e^{\beta\mu}}{1-e^{\beta\mu}} = \frac{1}{e^{-\beta\mu}-1} = n_B(\varepsilon = 0)$$

- μ wird Null im thermodyn. Limes $V \longrightarrow \infty$

$\Rightarrow$ Besetzungszahl $n \longrightarrow \infty$

$$\lim_{V \to \infty} \frac{1}{(e^{-\beta\mu} - 1)V} = n_0$$ (Teilchen im Grundzustand)

- kritische Dichte n_c bei Phasenübergang (T = const)
- oder kritische Temperatur T_c - " - (n = const)
- Ordnungsparameter $\frac{n_0}{n}$

$T_c \sim n^{\frac{2}{3}}$ $\qquad$ $p \sim T_c^{\frac{5}{2}} \sim n^{\frac{5}{3}}$