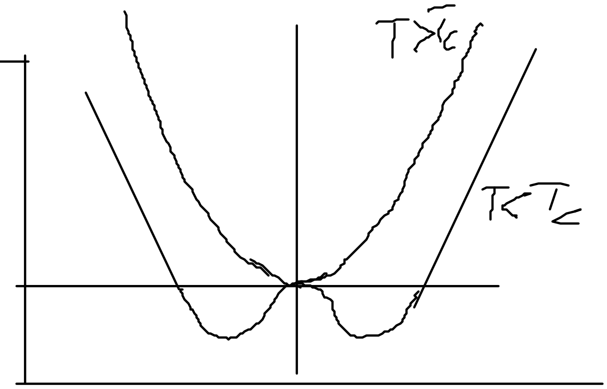
Landau-Theorie der Phasenübergänge (2. Art)

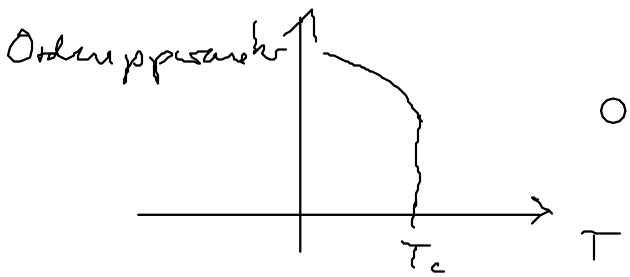
$$G_{var}(T, \vec{H}, \vec{m}) \xrightarrow[\min]{} \frac{\partial G}{\partial m} = 0 \Rightarrow \vec{m}_0(T, \vec{H}) \Rightarrow G(T, \vec{H}, \vec{m}_0(T, \vec{H}))$$

$T \sim T_c$: $G_{var} = a(T)\, \vec{m}^2 + b(T)\, \vec{m}^4$

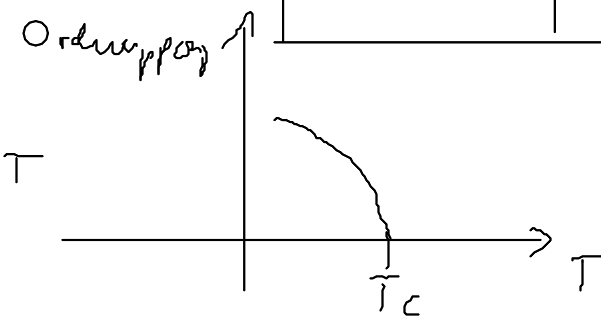


Phasenübergang 1. Art

m: Ordnungsparameter

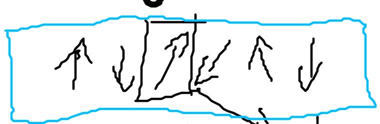


Phasenübergang 2. Art



Coarse graining

Beispiel



$m(\vec{r}) = \sum_{i \in \text{Region}} \vec{\sigma}_i$ Schwankungen klein

$\langle \sigma \rangle$ starke Schwankungen

$Z = \sum_n e^{-\beta E_n}$ bei $m(\vec{r})$ fest gibt es $n(m(\vec{r}))$ Zustände

$= \sum_{m(\vec{r})} \sum_{n(m(\vec{r}))} e^{-\beta E_n}$ mit Ordnungsparameter $m(\vec{r})$

$= \sum_{m(\vec{r})} \mathrm{Tr}_{m(\vec{r})}\left(e^{-\beta H}\right) = \int D m \; \mathrm{Tr}_{m(\vec{r})}\left(e^{-\beta H}\right) = \int Dm \; e^{-\beta F(m)}$

$$\boxed{\mathrm{Tr}_m\left(e^{-\beta H}\right) = e^{-\beta F(m)} := Z_m}$$

$$\boxed{F(m) = -\frac{1}{\beta} \ln\left(\mathrm{Tr}_m\left(e^{-\beta H}\right)\right) = -k_B T \ln Z_m}$$

- $F(m)$: Landau-freie-Energie-Funktional

$Z = \sum_m e^{-\beta F(m)}$ $\quad \left(\Longleftrightarrow \quad Z = \sum_n e^{-\beta H} \right)$

$$F(\{\vec{m}(\vec{r})\}, T, \vec{H}) = \int d^3\vec{r} \; f(m(\vec{r}), T, \vec{H})$$

$$f(\vec{r}) = f_N + f_0 \left(\frac{a(T)}{2} m(\vec{r})^2 + \frac{b(T)}{4} m(\vec{r})^4 + \frac{1}{2} \xi_0^2 \left|\vec{\nabla}_r m\right|^2 \right) - h m$$

angelegtes Feld

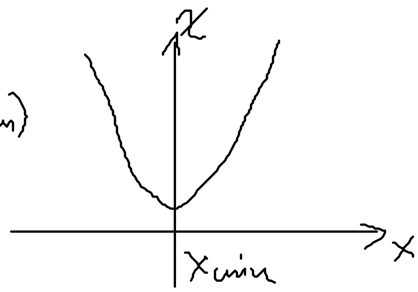
$f_N = f(m = 0)$

Entwicklung m klein: $f = f_N + \frac{a}{2} m^2 + \frac{b}{4} m^4 + |\nabla m|^2$

Molekularfeld-Näherung

$\int dx\, e^{\chi(x)}$

$\approx e^{-\chi(x_{min})}$



$Z_{MF} = e^{-\beta F(m_{min})}$

$m_{min} = ?$

Variationsrechnung:

$$\delta F = \int dV \Big(f_0 \big(a(T)\, m(r)\, \delta m(r) + b(T)\, m(r)^3\, \delta m(r) - \xi_0^2 \nabla^2 m(r)\, \delta m(r) \big) - h\, \delta m \Big)$$

$$\boxed{\delta(\nabla m \nabla m) = \nabla \delta m \nabla m + \nabla m \nabla \delta m = 2 \nabla \delta m \nabla m}$$

$$\delta F = \int dV (\dots)\, \delta m \Rightarrow f_0 (am + bm^3 - \xi_0^2 \nabla^2 m) - h = 0 \Rightarrow m_{min}$$

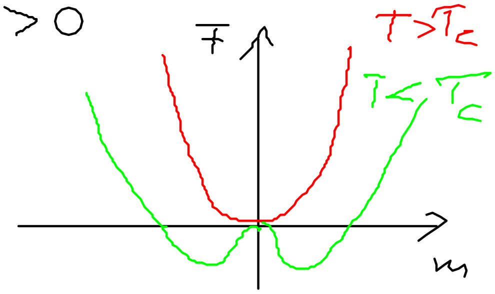
Homogene Lösungen

$\nabla^2 m = 0$ $\qquad am + bm^3 = \frac{h}{f_0}$

$a = \varepsilon = \frac{T - T_c}{T_c}$ $\quad (T \sim T_c)$ $\qquad b = \text{const} > 0$

- $h = 0$: $T > T_c$ $\qquad m = 0$

  $T < T_c$ $\qquad m = \pm \sqrt{-\frac{a}{b}} = \pm \sqrt{\frac{|\varepsilon|}{b}}$



Wärmekapazität

$Z = e^{-\beta F(m_{min})} = e^{-\beta G(T,H)}$ $\qquad G = F(m_{min})$ $\qquad C_H = -T \frac{\partial^2 G}{\partial T}$

$F = V f_0 \left(\frac{a}{2} m^2 + \frac{b}{4} m^4 \right) - V h m + V f_N$ $\qquad C_N = -T \frac{\partial^2}{\partial T^2} (V f_N)$

- $h = 0$: $T > T_c$ $\qquad F = V f_N(T)$

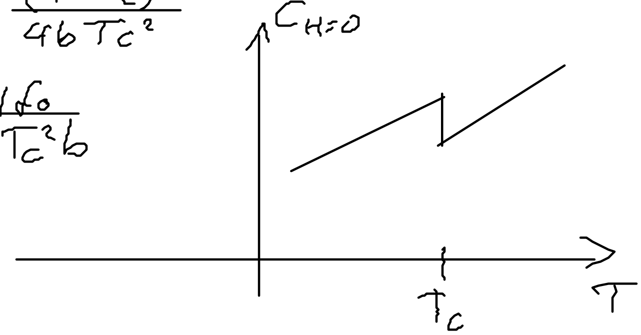
  $T < T_c$ $\qquad F = V f_0 \left(\frac{a}{2} \left(-\frac{a}{b} \right) + \frac{b}{4} \left(\frac{a^2}{b^2} \right) \right) + V f_N$

  $= V f_N(T) - V f_0 \frac{a^2}{4b}$

  $= V f_N(T) - V f_0 \frac{(T - T_c)^2}{4b T_c^2}$

$T > T_c$ $\qquad C_{H=0} = C_N + T \frac{V f_0}{2 T_c^2 b}$

$T < T_c$ $\qquad C_{H=0} = C_N$

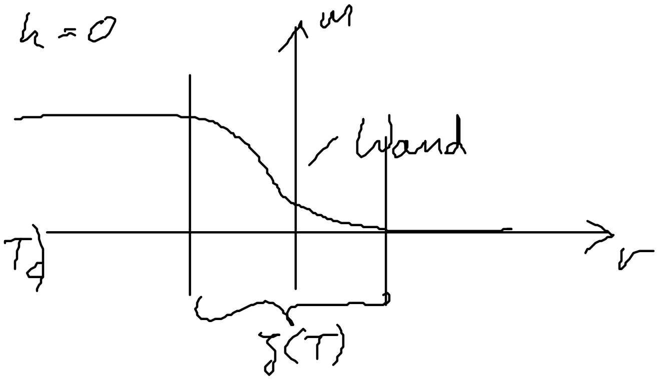


Nicht homogene Lösungen bei $h=0$

$am + bm^3 - \xi_0^2 \nabla^2 m = 0$

$m = m_0 \tanh\left(\frac{r}{2\xi(T)}\right)$

mit $\xi(T) = \xi_0 \frac{1}{\sqrt{2|a|}}$ $(a<0, T<T_c)$



$\xi(T)$: Korrelationslänge

bei $T \to T_c$: $a = \frac{T-T_c}{T_c} \to 0$ $\xi(T)$ divergiert

die Wand wird so groß