# Fluktuations - Dissipationstheorem

Kubo - Formel $V = -\int d\vec{r}^3\, F(\vec{r}, t)\, \hat{A}(\vec{r})$

$$\langle \delta B(\vec{r}, t) \rangle = \int dt'\, d\vec{r}'^3\, \chi(\vec{r}-\vec{r}', t-t')\, F(\vec{r}', t')$$

$$\chi_{BA}(\vec{r}-\vec{r}', t-t') = \frac{i}{\hbar} \operatorname{Tr}\left\{ \rho_0 \left[ B_I(\vec{r}, t), A_I(\vec{r}', t') \right] \theta(t-t') \right.$$

$$= \frac{i}{\hbar} \langle [ B(\vec{r}, t), A(\vec{r}', t') ] \rangle_0\, \theta(t-t')$$

$$\rho_0 = \frac{1}{Z} e^{-\frac{H_0}{kT}} = \frac{1}{Z} e^{-\beta H_0}$$

Heisenberg

Operatoren im Gleichgewicht

$$B = B_I = B_H \quad \text{wenn} \quad H = H_0$$

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Fall $A = B$ $\qquad H_0 |n\rangle = E_n |n\rangle$, $|n\rangle$ vollständig

$$\chi_{AA} = \frac{i}{\hbar} \frac{\theta(t-t')}{Z_0} \sum_n \langle n | e^{-\beta H_0} [A(t), A(t')] | n \rangle$$

$$A_H(t) = e^{i \frac{H_0 t}{\hbar}} A_S\, e^{-i \frac{H_0 t}{\hbar}}$$

$$= \frac{i}{\hbar} \frac{\theta(t-t')}{Z_0} \sum_n \Big( \langle n | e^{-\beta H_0} e^{i\frac{H_0 t}{\hbar}} A_S(r)\, e^{-i\frac{H_0 t}{\hbar}} e^{i\frac{H_0 t'}{\hbar}} A_S(r')\, e^{-i\frac{H_0 t'}{\hbar}} | n \rangle$$

$$- \langle \begin{smallmatrix} r \leftrightarrow r' \\ t \leftrightarrow t' \end{smallmatrix} \rangle \Big)$$

$$\sum_{n'} |n'\rangle\langle n'|$$

$$= \frac{i}{\hbar} \frac{\theta(t-t')}{Z_0} \sum_{n,n'} \langle n | e^{-\beta E_n} e^{i\frac{E_n t}{\hbar}} A_S(r)\, e^{-i\frac{E_{n'} t}{\hbar}} | n' \rangle$$

$$\langle n' | e^{i\frac{E_{n'} t'}{\hbar}} A_S(r')\, e^{-i\frac{E_n t'}{\hbar}} | n \rangle - \langle \begin{smallmatrix} r \leftrightarrow r' \\ t \leftrightarrow t' \end{smallmatrix} \rangle$$

$$= \frac{i}{\hbar} \frac{\Theta(t-t')}{Z_0} \sum \langle n|A(r)|n'\rangle\langle n'|A(r')|n\rangle e^{-\beta E_n} e^{\frac{i}{\hbar} E_n(t-t')} e^{-\frac{i}{\hbar} E_{n'}(t-t')}$$

$$- \quad \begin{matrix} r \leftrightarrow r' \\ t \leftrightarrow t' \end{matrix}$$

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FT $t \to \omega$ $$\int_0^\infty dt\, e^{i\omega t} e^{-\delta t} = \frac{1}{i\omega+\delta} = i \frac{1}{\omega + i\delta}$$

$$\overset{\delta \to 0}{=} i\left( PV \frac{1}{\omega} - i\pi\delta(\omega)\right)$$

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$$\chi(r, r', \omega) = \int_{-\infty}^{\infty} d\tau\, \chi(r, r', \tau) e^{i\omega\tau}$$

$\tau = t - t'$

$$= \frac{i}{\hbar} \frac{1}{Z_0} \sum_{n,n'} \langle n| A(r)|n'\rangle\langle n'|A(r')|n\rangle \frac{i\, e^{-\beta E_n}}{\omega + \frac{E_n - E_{n'}}{\hbar} + i\delta}$$

$$- \langle n|A(r')|n'\rangle\langle n'|A(r)|n\rangle \frac{i\, e^{-\beta E_n}}{\omega + \frac{E_{n'} - E_n}{\hbar} + i\delta}$$

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$$\chi(k, \omega) = \int d^3(r-r')\, \chi(r-r', \omega)\, e^{-ik(r-r')}$$

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wähle $r = r' \Rightarrow A(r) = A(r') = A$

$$\chi(\omega) = -\frac{1}{\hbar Z_0} \sum_{n,n'} |\langle n|A|n'\rangle|^2 e^{-\beta E_n}$$

$$\left( \frac{1}{\omega + \frac{E_n - E_{n'}}{\hbar} + i\delta} - \frac{1}{\omega - \frac{E_n - E_{n'}}{\hbar} + i\delta} \right)$$

Def. $\chi = \chi' + i\chi''$ $\quad \chi' = \mathrm{Re}(\chi) \quad \chi'' = \mathrm{Im}(\chi)$

$$\chi'' = \frac{\pi}{\hbar Z_0} \sum_{n,n'} |\langle n|A|n'\rangle|^2 \left\{ \delta\left(\omega + \frac{E_n - E_{n'}}{\hbar}\right) - \delta\left(\omega - \frac{E_n - E_{n'}}{\hbar}\right) e^{-\beta E_n} \right.$$

$$= \frac{\pi}{\hbar Z_0}\Big(\sum_{n,n'} |\langle n|A|n'\rangle|^2\, \delta\big(\omega + \frac{E_n - E_{n'}}{\hbar}\big)\, e^{-\beta E_n}$$

$$+ \sum_{n',n} |\langle n'|A|n\rangle|^2\, \delta\big(\omega + \frac{E_{n'} - E_n}{\hbar}\big)\, e^{-\beta E_{n'}}\Big)$$

$$= \frac{\pi}{\hbar Z_0} \sum_{n,n'} |\langle n|A|n'\rangle|^2\, \delta\big(\omega + \frac{E_n - E_{n'}}{\hbar}\big)\, \big(e^{-\beta E_n} - e^{-\beta E_{n'}}\big)$$

$$\frac{E_n - E_{n'}}{\hbar} = -\omega \implies E_{n'} = E_n + \hbar\omega$$

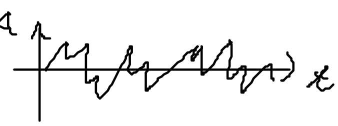
$$\boxed{\chi'' = \frac{\pi\, e^{-\beta E_n}\,(1 - e^{-\beta\hbar\omega})}{\hbar Z_0} \sum_{n,n'} |\langle n|A|n'\rangle|^2\, \delta\big(\omega + \frac{E_n - E_{n'}}{\hbar}\big)}$$

$$\langle A(t) A(t')\rangle_0 = \mathrm{Tr}\,(\varrho\, A(t)\, A(t'))$$ Korrelationsfunktion

$$S(t-t') = \frac{1}{2}\langle A(t) A(t') + A(t') A(t)\rangle_0$$

Symmetrisierte Korrelationsfunktion - Rauschen

$\langle A\rangle = 0$ aber




wie charakterisiert man A ? (das Rauschen)

$\tau = |t - t'|$ Korrelation klein, wenn $A(t')$ unabhängig von $A(t)$

$$S(t-t') = \frac{1}{2Z_0}\sum_n \Big(\langle n|e^{-\beta H_0} e^{i\frac{H_0 t}{\hbar}} A\, e^{-i\frac{H_0 t}{\hbar}} e^{\frac{i}{\hbar}H_0 t'} A\, e^{-\frac{i}{\hbar}H_0 t'}|n\rangle$$

$$+ \; t \leftrightarrow t' \Big)$$

$$= \frac{1}{2Z_0}\sum_{n,n'} |\langle n|A|n'\rangle|^2 e^{-\beta E_n} e^{\frac{i}{\hbar}(E_n - E_{n'})(t-t')} + t \leftrightarrow t'$$

FT

$$S(\omega) = \frac{1}{2Z_0} \sum_{n,n'} |\langle n|A|n'\rangle|^2 e^{-\beta E_n} \left[ 2\pi \left( \delta\left(\omega + \frac{E_n - E_{n'}}{\hbar}\right) + \delta\left(\omega - \frac{E_n + E_{n'}}{\hbar}\right) \right) \right]$$

$$\int dt\, e^{i\omega t} = 2\pi\, \delta(\omega)$$

$$= \frac{\pi}{Z_0} e^{-\beta E_n} \left(1 + e^{-\beta\hbar\omega}\right) \sum_{n,n'} |\langle n|A|n'\rangle|^2 \, \delta\left(\omega + \frac{E_n - E_{n'}}{\hbar}\right)$$

$$\boxed{S(\omega) = \hbar \coth\left(\frac{\beta\hbar\omega}{2}\right) \chi''(\omega)}$$

Klassischer Limes

$\coth(x) \approx \frac{1}{x}$ $\qquad x \ll 1$

für $k_B T \gg \hbar\omega$

$$S(\omega) \approx \frac{2}{\omega\beta} \chi''(\omega) = \frac{2 k_B T}{\omega} \chi''(\omega)$$

Quantenrauschen $\longleftrightarrow$ klass. Rauschen

$\hbar\omega \gtrsim k_B T$ $\qquad\qquad$ $\hbar\omega \ll k_B T$

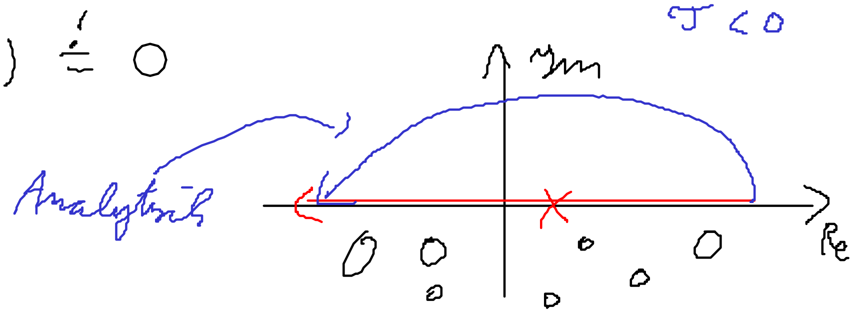
$\chi(t-t') \propto \Theta(t-t') \Rightarrow \chi(\omega)$ Analytisch für $\mathrm{Im}(\omega) > 0$

$$\chi(\tau) = \int \frac{d\omega}{2\pi} e^{-i\omega\tau} \chi(\omega)$$

$\omega = i\alpha \Rightarrow e^{\alpha\tau}$

$\tau < 0$

mit $\chi(\tau < 0) \overset{!}{=} 0$



$$\chi'(\omega) = \frac{PV}{\pi} \int d\omega' \, \frac{\chi''(\omega)}{\omega' - \omega}$$

$$\chi''(\omega) = -\frac{PV}{\pi} \int d\omega' \, \frac{\chi'(\omega)}{\omega' - \omega}$$

Kramers Kronig Relation